Texas Essential Knowledge and Skills (TEKS)

Math 5.4B Algebraic reasoning
The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:
represent and solve multi-step problems involving the four operations with whole numbers using equations with a letter standing for the unknown quantity;

Math 6.9A Expressions, equations, and relationships.
The student applies mathematical process standards to use equations and inequalities to represent situations. The student is expected to:
write corresponding real-world problems given one-variable, one-step equations or inequalities

Objectives
1. Students will learn vocabulary related to space figures.
2. Students will be able to name 3-dimensional objects.
3. Students will be able to identify parts of 3-dimensional objects.
4. Students will write algebraic expressions that help find the amount of faces, edges, or vertices of a particular polyhedron.

Materials
The Third Dimension worksheet
Internet access (for looking up definitions)
Classifying Polyhedrons worksheet
Notecards labeled 1-8 (for numbering 8 different stations)
1 Rubik’s® Cube
7 Shape blocks (preferably: a regular tetrahedron, rectangular pyramid, triangular prism, hexagonal prism, regular octahedron, regular dodecahedron, and regular icosahedron)
Writing Rules worksheet

www.YouCanDoTheCube.com
Procedure

1. Students fill out The Third Dimension worksheet by searching online for definitions.

2. While students are working, set up 8 stations around the classroom by placing one numbered notecard and one shape at each location (the Rubik’s Cube counts as one of the 8 shapes).

3. When done with the worksheet, students share and compare the definitions they found, and discuss the similarities and differences of their findings.

4. As students get out their Classifying Polyhedrons worksheet, explain to them that they will be visiting each station in an attempt to name each shape (using two words) and determine how many faces, edges, and vertices it is comprised of.

5. Spread the students out at the different stations and let them start filling out their worksheet, writing down their answers for each station in the corresponding numbered row.

6. Students compare their answers and compile what they think the answer key is; and then compare to the actual key.

7. Students work on their Writing Rules worksheet. They will focus on specific groups of polyhedrons when counting faces, edges, and vertices to discover shortcuts in the counting process. Then they will translate those shortcuts into algebraic expressions.

Notes to Teacher

- Depending on the length of your class, good breaks would be after procedure 5, or after procedures 3 and 6.

- My students tend to struggle with naming 3-dimensional shapes while visiting the stations. I encourage them to make educated guesses based on the definitions that we have been studying.

- You will need to create your own answer key for the Classifying Polyhedrons worksheet, due to the fact that you may use different shapes, more or less shapes, or put shapes at different stations.

- During the Writing Rules worksheet, give students access to as many of the shape blocks as you can. It gives them a hands---on resource while they are trying to develop hypotheses and test theories.
The Third Dimension

Define the following words.

Space figure – A 3-dimensional shape that has depth in addition to length and width

Polyhedron – A 3-dimensional shape made up of polygons (no curves)

Parts of a Polyhedron: Face – A flat surface of a 3-dimensional shape

Edge – A line segment that connects two faces

Vertex – A point where the edges meet

Name the shape that is both a prism and a platonic solid:

Cube
(Regular Hexahedron)
(Rectangular Prism)

Name the shape that is both a pyramid and a platonic solid:

Regular Tetrahedron
(Triangular Pyramid)
Classifying Polyhedrons

Visit each of the shape stations. At each station, attempt to correctly name each space figure (using two words). Then document the number of faces, edges, and vertices of each shape. Leave the last column blank; it will be used later in the lesson.

<table>
<thead>
<tr>
<th>Polyhedron</th>
<th># of Faces</th>
<th># of Edges</th>
<th># of Vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
Prisms
1) Count the faces, edges, and vertices of the following prisms.

<table>
<thead>
<tr>
<th>Name</th>
<th>n</th>
<th>faces</th>
<th>vertices</th>
<th>edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>triangular prism</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>rectangular prism</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>pentagonal prism</td>
<td>5</td>
<td>7</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

*n represents the number of sides of one of the bases

Take n and: +2  x2  x3

2) Find the patterns in the table above and use it to make a prediction.

<table>
<thead>
<tr>
<th>Name</th>
<th>n</th>
<th>faces</th>
<th>vertices</th>
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</tr>
</thead>
<tbody>
<tr>
<td>hexagonal prism</td>
<td>6</td>
<td>8</td>
<td>12</td>
<td>18</td>
</tr>
</tbody>
</table>

3) Draw a hexagonal prism and check your prediction.

4) Write function rules that describe the shortcuts in finding the number of faces, edges, and vertices of a prism. (vertices is already completed as an example)

Faces: \( F = n + 2 \)

Edges: \( E = 3n \) or \( E = 3 \cdot n \)

Vertices: \( V = 2n \) or \( V = 2 \cdot n \)

Pyramids
1) Count the faces, edges, and vertices of the following pyramids.

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<td>4</td>
<td>6</td>
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*n represents the number of sides of the base

Take n and: +1  +1  x2
2) Find the patterns in the previous table and use it to make a prediction.

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3) Draw a hexagonal pyramid and check your prediction.

4) Write function rules that describe the shortcuts in finding the number of faces, edges, and vertices of a pyramid.

   Faces: $F = n + 1$
   Edges: $E = 2n$
   Vertices: $V = n + 1$

Platonic Solids

What shortcuts could be used to count the faces, edges, and vertices of these regular polyhedrons? Write a description and try it.

For the dodecahedron on the right, there are 12 faces. Each face is a pentagon, which has 5 edges (sides). $5 \times 12 = 60$; however an edge connects two faces, so we are counting each edge twice. $60/2 = 30$; so there are 30 edges. Each face also has 5 vertices (angles). $5 \times 12 = 60$; however a vertex connects three edges of three different faces. $60/3 = 20$; so there are 20 vertices.

Euler’s Formula

Go back to the Classifying Polyhedrons worksheet. Label the blank column “$F + V$“, and in each row add the number of faces to the number of vertices and record that in the new column. When you get done with that, compare column “$F + V$“ with column “$E$“, and write down a function rule describing the pattern.

$F + V = E + 2$  
this formula works for every polyhedron
The Third Dimension

Define the following words.

Space figure – __________________________________________________________

_______________________________________________________________________

Polyhedron – __________________________________________________________

_______________________________________________________________________

Parts of a Polyhedron:  Face – _____________________________________________

_______________________________________________________________________

Edge – ________________________________________________________________

_______________________________________________________________________

Vertex – ______________________________________________________________

_______________________________________________________________________

Special Polyhedrons

Prism -  ____________________________  Platonic Solid -  ____________________________  Pyramid -  ____________________________  Others:  ____________________________

Concave, Truncated, etc...

_______________________________________________________________________

Name the shape that is both a prism and a platonic solid:

______________________________

Name the shape that is both a pyramid and a platonic solid:

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Classifying Polyhedrons

Visit each of the shape stations. At each station, attempt to correctly name each space figure (using two words). Then document the number of faces, edges, and vertices of each shape. Leave the last column blank. It will be used later in the lesson.
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*\( n \) represents the number of sides of one of the bases

2) Find the patterns in the table above and use it to make a prediction.

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3) Draw a hexagonal prism and check your prediction.

4) Write function rules that describe the shortcuts in finding the number of faces, edges, and vertices of a prism. (Vertices is already completed as an example)

Faces: \( F = \)

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