How to Create Your Own Rubik’s Cube Mosaic
Using GIMP, a Free Photo Editing Program
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Materials

Introduction
As you can see in the picture above, a mosaic can be made by using an array of Rubik’s cubes. Each tile on one face of a cube acts as a “pixel” (9 pixels for each 3 x 3 Rubik’s Cube) and each cube gives you 6 colors in your palette to choose from. There are many different ways a mosaic can be created, and there are many variations that can arise when creating one. This guide was created by a teacher and students that were new to this process for others that are new to the process, so this not intended to be comprehensive, but instead to give an introduction to mosaics and how to do a very basic conversion. Good luck!

Challenges you may encounter:
- Your picture has too many pixels: A 15 Megapixel camera will give you a maximum picture resolution of 4752 x 3168 or over 15 million pixels! Chances are you aren’t going to have enough cubes to create your mosaic exactly like your picture.
- Your picture has too many colors: The Rubik’s cube only has a 6 color palette so you will only have 6 colors to work with.

However, even with these challenges, you can still have fun and create nice mosaics – it just might take a little time to get it just right. The You CAN Do the Rubik’s Cube Program will lend you sets of cubes to create Mosaics for an 6 week period for FREE. http://www.youcandothecube.com/cube-mosaics/
Step 1 – Let’s cube some 2d math!

You could take all your Rubik’s cubes and make a square and make your picture fit into that square, but that’s adding an unnecessary limitation on top of those we just talked about. Doing a little bit of math we can come up with a list of possible mosaic dimensions (the height and width). Once we have a picture, we can select the dimensions that most closely match.

Analyzing the mosaic

How many cubes you will be working with will lead you to a number of choices for creating your mosaic. The first thing we want to know is how many pixels we have to work with.

\[
\begin{align*}
\text{c: cubes} & \quad 9c = p_t \\
\text{p}_t: \text{ pixels (total)} & \\
\end{align*}
\]

You can now look at the range of lengths and widths you have to choose from, however, you can’t choose just any height and width that give a product equal to your total pixels. Our example above had 2025 pixels (225 Rubik’s Cubes). You cannot make a mosaic that is 81 pixels wide by 25 pixels tall. Remember that when actually building the mosaic you are placing 1 cube at a time, which is 9 pixels at a time. This also means the height and width go up by 3 pixels at a time. The height of 25 is not evenly divisible by 3, so we would not have enough cubes. For now, it will be simpler to keep our units for area as cubes instead of pixels.

Generate a list of mosaic dimensions

A good starting point is to take the square root of the number of cubes available and then round down to get the largest square that can be made. Our example of 200 cubes would give a result of \( \sqrt{200} \approx 14 \). This gives you a starting point of a square made of 14 \( \times \) 14 Rubik’s Cubes. You can decrease one dimension incrementally—just divide and round down.

Once you have this list you can then multiply to get the height and width in pixels:

\[
\begin{align*}
\sqrt{225} & \approx 15 \Rightarrow 15 \text{ cubes} \times 15 \text{ cubes} & x3 & \Rightarrow 45 \text{ pixels} \times 45 \text{ pixels} \\
225/14 & \approx 16 \Rightarrow 14 \text{ cubes} \times 16 \text{ cubes} & x3 & \Rightarrow 42 \text{ pixels} \times 48 \text{ pixels} \\
225/13 & \approx 17 \Rightarrow 13 \text{ cubes} \times 17 \text{ cubes} & x3 & \Rightarrow 39 \text{ pixels} \times 51 \text{ pixels} \\
200/12 & \approx 18 \Rightarrow 12 \text{ cubes} \times 18 \text{ cubes} & x3 & \Rightarrow 36 \text{ pixels} \times 54 \text{ pixels} & \text{etc.}
\end{align*}
\]

Another way to generate this list is using a spreadsheet. Rubik-Mosaic-Helper is an Excel spreadsheet that was created to generate this list with the only input being the number of cubes. The spreadsheet is completely unlocked to encourage users to look at the formulas used in the cells and look at the logic involved to see how these lists are created.

Analyzing a picture

A picture’s resolution is usually given in pixels as height \( \times \) width. We now want to see the resolution in total pixels to compare to our mosaic constraints.

\[
\begin{align*}
\text{h}_p: \text{ height (in pixels)} & \quad 9h_p = p_t \\
\text{w}_p: \text{ width (in pixels)} & \\
\end{align*}
\]

Example: Your picture is 1600x1200 => \( h_p = 1600 \), \( w_p = 1200 \)

\[
\begin{align*}
h_p w_p = p_t \\
(1600)(1200) = p_t \\
so, \text{the picture is} 1,920,000 \text{ pixels} \Rightarrow 1,920,000 = p_t
\end{align*}
\]

This is why we came up with the total pixels number for the mosaic earlier. You can now compare the pixels we need to what we actually have. The picture has almost 1000 times more pixels than our mosaic!
Step 2 – Smart image selection

This step becomes much more important if the number of cubes available to you is smaller. The general rule of thumb here is to find a picture that coincides with the 2 main challenges of a Rubik’s Mosaic: 1) the fewer the number of colors, the better, and 2) the fewer the number of total pixels the better. It was through trial and error that we found image selection made the biggest difference in getting a good final result. Your results may differ, so play around! The examples shown below will actually use a different image than that which we used to make our final mosaic to better highlight some of the difficulties that arise from selecting a more difficult starting image.

Step 3 – Image manipulation

I will explain some tools that are available using an image manipulation program called GIMP (GNU image manipulation program), which anyone can download for free. [http://www.gimp.org/](http://www.gimp.org/)

I used version 2.6.11. If you use a different version you might need to search around for the same features. If you have never done image manipulation you might need to refer to the GIMP user manual, since this document is focusing on the lesser known features that are useful for Rubik’s mosaics.

**Instructions on what to do within the GIMP program are indented from the rest of the text.**

Open your target picture in GIMP:

- Go to File → Open
- Select your image

**Cropping**

First make sure the image you’ve selected is as small as possible. In our example we started with a 1600 wide x 1200 tall picture, but decided to focus just on the horse’s head, and cropped to 320 wide x 830 tall. Your new pixel size can be put into the mosaic helper spreadsheet.

**In the Toolbox:**

- Select the “Rectangle Select Tool”

**In the Image window:**

- Select the portion of the picture you would like to crop down to.
- Click to open the “Image” Menu
- Click “Crop to Selection”
Resolution
At this point you have a target image that is likely still much larger than the number of Rubik’s pixels available. This means you have to decrease the number of pixels making up your picture. Note that image processing programs will likely use the term “resolution” to mean pixels per inch, and “image size” to mean total pixels. The pixels per inch setting only matters when printing your digital photo, which we won’t be doing. When using the term resolution we are referring to the total number of pixels.

Scale
You now should have a list of possible height to width ratios and their pixels (either on your own or using the provided spreadsheet). The Rubik’s height to width ratio that most closely matches the picture’s ratio is now chosen, and the corresponding pixels are now used to set the pictures height and width. You are selecting only one number to use based on this logic:

If (Picture height:width ratio) > (Rubik’s Cube height:width ratio), then (use Height), Else (use Width).

If you used the spreadsheet, column H will tell you which Rubik’s pixel number to use when setting this.

In GIMP, select “Image”, then “Scale Image” to get the scale image dialog window. Note the blue oval is highlighting a chain icon. This icon represents whether or not the ratio of height to weight should be fixed. The default is fixed. If you click this the icon changes to a broken chain, and this means that your final image will be more distorted. The trade off here is distortion vs lost resolution, and we have usually chosen decreased resolution, but your results may vary. The example will use a fixed ratio to minimize distortion.

Click the number in the “Image Size” field that corresponds with whichever dimension you have just determined to use (our example is Width = 27). Press the tab key to select another field, and you should see the other dimension update to keep height to width as close to the original ratio as possible (in this example our height is now 70 instead of 75 – this shows how much resolution you have just lost). Click “Scale” and your image is now very small. In the bottom of the image window you can make the percentage larger to zoom in. Your image is now the correct number of total pixels!
Color Palette

The Rubik’s Cube has 6 colors White, Yellow, Blue, Green, Red, and Orange. We are going to convert our picture to only use these 6 colors.

Levels

Click “Colors”, then “Levels...” to open the Levels dialog window: Input Levels in this window is actually one of the most valuable tools to use for any image manipulation. In this window you can fix a number of mistakes made when a picture was taken. In the example to the right, you are looking at a graphical representation of shadows, highlights, and midtones. The X axis is the scale from black (0) to white (255), and the Y axis is the number of pixels at each particular level. In the example, we can see that there are a lot of pixels in this picture that are white, or very nearly white due to the larger Y values at the far right of the graph. There are 3 settings here that we are going to adjust: Black level (also called black point or shadows), White level (also called white point or highlights), and Mid tone (also called midpoint or gamma). Black level takes the value you have selected and makes all pixels equal or below that value black. If you slide from 0 all the way up towards 255 you will see the picture fade to black. White level works the same: all values above it become white. As you move black and white levels closer to each other, you are removing a lot of colors (recall that our goal here is to eventually have only 6 colors). The gamma (or mid point) setting changes from lighter/more color to darker/less color as you go left to right. Changing all of these values would also be equivalent to changing contrast and brightness, but within the levels screen you have much more control over the settings. You should now spend a fair amount of time playing with these values and watching how they change the image. **The only rule to follow here is you must bring black level and white level much closer together, but where they are on the chart will vary for different pictures.** For a lot of pictures this step will make the background simply disappear. If instead your background is blending into the picture in a bad way, you might need to selectively delete it. Here is where we ended up on our example, but there are many other choices for levels that also could have worked, but just looked different:
Color Mode

While the number of colors has been drastically reduced already, we are now going to fix the number at 5 or 6. To do this we are going to use a color mode called “indexed color”. This means we select the exact colors to use and GIMP will change each pixel to its nearest color in the palette.

5 or 6 ???

It turns out that for a LOT of the pictures we have wanted to convert, the green color on the Rubik’s Cube doesn’t blend as well: White through red blend very well and red to blue typically does ok. However, blue to green often doesn’t look very good.

Once you try this a few times, it’s easy to switch colors around quickly.

Switch to Indexed

Click “Image”, then “Mode”, then “Indexed...” to open the “Indexed Color Conversion” dialog window. Click “Generate optimum palette”, and enter “6” for maximum number of colors.

Note the “Dithering” setting is “None”. Dithering is when 2 colors are stitched together to give the illusion of another color or a gradient between the two. Here’s an example of how red and blue dithering can appear purple if the pixels are sufficiently small. You can play around with different dithering settings to see if you like them.
**Convert Optimized Indexed to Rubik’s Palette**

Click “Windows”, then “Dockable Dialogs”, then “Colormap”.

You can now see the 6 color palette that GIMP has selected based on the colors that were left after setting levels. This is another method of interpolation; however, GIMP doesn’t allow you to have any say in the implementation as they do in other situations (like we saw in scaling).

Right click on any color, and select “Edit color…” . There are many different methods available to select the Rubik’s Colors. Here are some example HTML codes for them:

<table>
<thead>
<tr>
<th>Color</th>
<th>HTML Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>#ffffff</td>
</tr>
<tr>
<td>Yellow</td>
<td>#fff00</td>
</tr>
<tr>
<td>Orange</td>
<td>#ff9c00</td>
</tr>
<tr>
<td>Red</td>
<td>#f60000</td>
</tr>
<tr>
<td>Blue</td>
<td>#3737b3</td>
</tr>
<tr>
<td>Green</td>
<td>#00d900</td>
</tr>
</tbody>
</table>

**To Green or Not to Green**

You can now play around with the colors and decide whether or not to use green. If you don’t want to, just repeat the last step and choose only 5 colors. If you are more advanced with image manipulation, you might try leaving the green and manually substituting dithering with other colors in the areas defined as green. Here are some examples we came up with: (The left image is AFTER indexing to 6, but obviously before converting to Rubik’s Palette)

![Image showing color selection](image)

**Which would you choose?**

We decided none of these were contenders to actually become a mosaic, but thought the 2nd image from the left turned out OK.

**Try Again?**

As you do this more, you will start to get much faster at this, and you will start to know whether an image will come out before you get there. One thing you must try with some image is to change up the order of manipulation. Since so much of your final image comes from some mysterious math being done by the computer, changing the order can dramatically alter the final product (for better or worse). If you decide to change up the order of manipulation crop really should always be first, and if you do Indexed before levels, just don’t do levels at all... Here’s another example of order:

**Crop (this should always be first) >> Levels >> Indexed >> Scale >> Convert**
Mosaic Time!

Building Instructions
Now you need to add some gridlines to your image, and decide on how many sections to break down into. Typically it will make it easier for builders if the grid sizes are smaller. It makes it more manageable to get built without errors. Then make digital copies that can be zoomed by the builders, or print out the grids. We opted to print a couple “big picture” copies and then a zoom in of each grid:

Building Logistics –
You’ve gone to all this effort to make an image, then you’re going to put forth a bunch more to actually build it. Please make sure you’ve thought about how people will see this before you get to that point! Here’s some thoughts/ideas:

1) Typically these will have more impact if people can view it from far away and up close.
2) Think about making a stand for your mosaic. In the example picture at the beginning of the document you can see some wood framing. This was custom made for this project. The entire assembly could be tipped up to about 10 degrees from vertical (totally vertical could make them fall!) to be seen from farther away.
3) Set up a camera on a tripod and take some video of the mosaic being made—then use some editing software to speed up that video to make the building process time-lapse!
4) Make an event out of the building process
   a. Find a sponsor—put some logo/ad behind where the cubes will go. The time-lapse will be extra fun then! A sponsor might also help you finance getting more cubes!
   b. Have lay people try to help solve cubes for the mosaic—it’s actually much easier than learning to solve a cube, since it’s only solving one layer.
Objective
Create Your Own Mosaic using 3 x 3 Rubik’s Cubes using GIMP photo editing program

Materials


The Standards for Mathematical Practice:

CCSS.Math.Practice.MP1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

CCSS.Math.Practice.MP2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

CCSS.Math.Practice.MP5 Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for
their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

National Education Technology Standards:

1. Creativity and Innovation
Students demonstrate creative thinking, construct knowledge, and develop innovative products and processes using technology.

   Students:
   a. apply existing knowledge to generate new ideas, products, or processes.
   b. create original works as a means of personal or group expression.

2. Communication and Collaboration
Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others. Students:
   a. interact, collaborate, and publish with peers, experts, or others employing a variety of digital environments and media.
   d. contribute to project teams to produce original works or solve problems.

6. Technology Operations and Concepts
Students demonstrate a sound understanding of technology concepts, systems, and operations. Students:

   Grade 6 EE. Reason about and solve one-variable equations and inequalities.
   CCSS.Math.Content.6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.

   Grade 7 EE. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
   CCSS.Math.Content.7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

   Grade 8 EE. Expressions and Equations
   CCSS.Math.Content.8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form x² = p and x³ = p, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.

   Modeling with Geometry HSG-MG
   Apply geometric concepts in modeling situations

   • CCSS.Math.Content.HSG-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

   • CCSS.Math.Content.HSG-MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

   • CCSS.Math.Content.HSG-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).