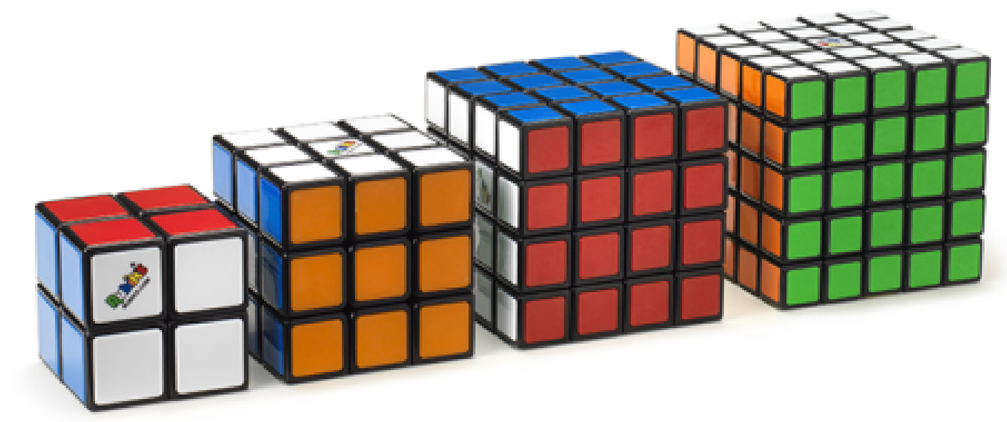
Deconstructing Prisms



Using Patterns, Write Expressions That Determine the Number of Unit Cubes With Any Given Number of Exposed Faces

Based on the work of Linda S. West, Center for Integrative Natural Science and Mathematics
Kentucky Center for Mathematics

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Deconstructing Prisms

Math Concepts & Standards

CCSS.MATH.CONTENT.5.OA.A.2
Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.
CCSS.MATH.CONTENT.6.EE.A.2.A
Write expressions that record operations with numbers and with letters standing for numbers.
CCSS.MATH.CONTENT.6.EE.B.6
Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
CCSS.MATH.CONTENT.7.G.B.6
Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Materials

- Rubik's® Cubes - sets of 27 cubes (one set for each student or group)
  - Cubes can be borrowed from www.youcandothecube.com/lending-library/
- Handouts for each student

In this activity students will:

- Use the vocabulary of prisms in recognizing patterns
- Extend patterns
- Describe the pattern that leads from one size to another
- Find an explicit rule for each pattern to predict the total number of cubes and/or exposed faces for any size cube (puzzle); recognize equivalent expressions
- Determine the size of the Rubik's® Cube (Puzzle) given the total number of unit cubes or the number of unit cubes with a given number of exposed faces
- Evaluate expressions

Vocabulary

- prism, regular rectangular prism
- bases
- faces, lateral faces
- unit cube
- edge
- corner (vertex, vertices)
- exposed

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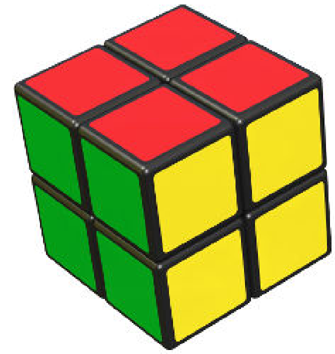
Rubik's® Cube Vocabulary



• The mathematical term for the Rubik's® Cube shown is **prism**. A prism has two **bases** and rectangular **faces**.

The top and bottom of a prism are called the **bases**. What shape are the bases of this prism?

The sides of a prism are called **lateral faces**, or sometimes just **faces**. The faces of a prisms are always parallelograms. In right prisms, the faces are rectangles. Why?

How many lateral faces are there?

What determines the number of lateral faces in a prism?

If the bases of a prism are regular polygons, the prism is called **regular**. The Rubik's® Cube is a regular rectangular prism. Explain why.

• The size of a Rubik's® Cube is determined by the number of smaller cubes or **unit cubes** on the **edge** of the larger cube. An edge is a line segment where exactly 2 faces meet. How many edges are there in the cube above?

What is the length of the edge of the cube above?

• A **corner** or **vertex** is the point where exactly 3 faces meet. How many corners or vertices are there in the cube above?

• Are there any unit cubes in the interior of this cube that can't be seen?

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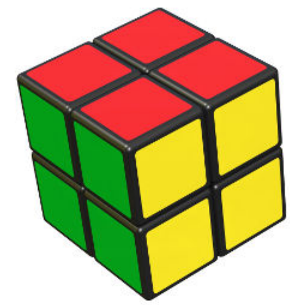
Teacher Notes: This page could be classwork after some general instruction to prisms. It reinforces the prism vocabulary using a 2x2x2 Rubik's® Cube. It could also be used as the instruction for prism vocabulary.

Rubik's® Cube Vocabulary

• The mathematical term for the Rubik's® Cube shown is **prism**. A prism has two **bases** and rectangular **faces**.



The top and bottom of a prism are called the **bases**. What shape are the bases of this prism? Square

The sides of a prism are called **lateral faces**, or sometimes just **faces**. The faces of a prisms are always parallelograms. In right prisms, the faces are rectangles. Why? It is assumed here that students are working with right prisms. Perhaps the student's working definition of a prism at this point is that the bases are parallel and segments connecting the vertices are perpendicular to the bases. Therefore, the lateral faces would have 4 right angles making them rectangles. It is important that students know the term **lateral faces** because there are times when the term faces will include the bases. This can be confusing even when using context clues. Using the term lateral faces helps to clarify.

How many lateral faces are there? 4 If this is being used for instruction, the teacher may also ask. "How many lateral faces there would be if the bases were triangular or some other polygon?".

What determines the number of lateral faces in a prism? Students should generalize that the number of lateral faces is equal to the number of sides on the base.

If the bases of a prism are regular polygons, the prism is called **regular**. The Rubik's® Cube is a regular rectangular prism. Explain why. The base of a Rubik's® Cube is a square. A square is a regular polygon because all the sides and angles are congruent. A square is a rectangle. A regular rectangular prism is also called a square prism. If all the faces, including the bases, are squares, the prism is a cube.

• The size of a Rubik's® Cube is determined by the number of smaller cubes or **unit cubes** on the **edge** of the larger cube. An edge is a line segment where exactly 2 faces meet. How many edges are there in the cube above? 12 Students may generalize that the number of lateral faces is equal to the number of sides of the base times 3.

What is the length of the edge of the cube above? 2 units The teacher may want to reinforce the difference between linear units (units of perimeter), square units (units of area), and cubic units (units of volume). An extension could be made to exponents and the vocabulary of squaring and cubing a number.

• A **corner** or **vertex** is the point where exactly 3 faces meet. How many corners or vertices are there in the cube above? 8 Students may generalize that the number of vertices is equal to the number of sides of the base times 2.

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• Are there any unit cubes in the interior of this cube that can’t be seen? No. Students could use smaller unit cubes to prove this. This sets up the work that follows which focuses on the number of exposed faces of the unit cubes which make up the larger cube.

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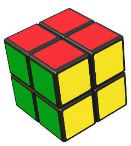
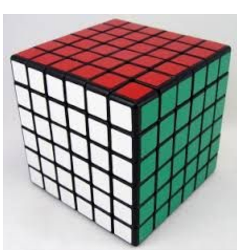
- Look for patterns in the number of faces that are exposed on the unit cubes. You will be counting the colored sides of each of the unit cubes.
- To help you write the rule, you may want to look at what you learned on the previous page.

Large Cube	Edge length of the large cube	Number of cubic units	3 faces exposed on unit cubes	2 faces exposed on unit cubes	1 face exposed on unit cubes	0 faces exposed on unit cubes
						
						
						
						
						
Rules: Is there a way to predict the number of cubes and faces based on the size of the edge of a cube?	*e*	Hint: How many squares on 1 edge?	(corners)	(edges)	(center)	Hint: How many cubes can't be seen?

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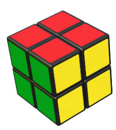
Teacher Notes: Students may work individually or in pairs to complete the chart. Remind students that they are focusing on the faces of the unit cubes. This will be important as they may confuse **edge of the prism** with **edge pieces** which have 2 exposed faces. The teacher may want to have unit cubes available so that students can build the first two or three cubes to see how faces of the unit cubes become "hidden" in the larger cube. Have students look for generalizations after completing each larger cube so they are working towards the rule throughout the task.

Look for patterns in the number of faces that are exposed on the unit cubes. You will be counting the colored sides of each of the unit cubes. To help you write the rule, you may want to look at what you learned on the previous page.

Large Cube	Edge length of the large cube	Number of cubic units	3 faces exposed on unit cubes	2 faces exposed on unit cubes	1 face exposed on unit cubes	0 faces exposed on unit cubes
	2	**8** 2·2·2 2^3	8 4 • 2	**0** 12(2-2) 12(0)	**0** $6(2-2)^2$ $6(0)^2$ 6(0)	**0** $(2-2)^3$ (0)3
	3	**27** 3·3·3 33	8 4 • 2	**12** 12(3-2) 12(1)	**6** $6(3-2)^2$ $6(1)^2$ 6(1)	**1** $(3-2)^3$ $(1)^3$
	4	**64** 4·4·4 43	8 4 • 2	**24** 12(4-2) 12(2)	**24** $6(4-2)^2$ $6(2)^2$ 6(4)	**8** $(4-2)^3$ $(2)^3$
	5	**125** 5·5·5 53	8 4 • 2	**36** 12(5-2) 12(3)	**54** $6(5-2)^2$ 6(3)2 6(9)	**27** $(5-2)^3$ (3)3
	6	**216** 6·6·6 63	8 4 • 2	**48** 12(6-2) 12(4)	**96** $6(6-2)^2$ $6(4)^2$ 6(16)	**64** $(6-2)^3$ $(4)^3$
Rules: Is there a way to predict the number of cubes and faces based on the size of the edge of a cube?	*e*	Hint: How many squares on 1 edge? e • e • e e^3	(corners) 8 2 • 4	(edge pieces) 12(e - 2) or 2•4(e-2)+4(e-2) or 3•4(e-2)	(center pieces) $6(e-2)^2$ or $(2+4)(e-2)^2$	Hint: How many cubes can't be seen? $(e-2)^3$

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Teacher Notes:

Have students share their rules. Have them explain how they formulated the rules so that they recognize equivalent expressions such as those for edge pieces.

Challenge: On the vocabulary page, students may have generalized about the number of corners and edges in a prism. Have students articulate why the 2, 12, and 6 remain constant in the corner, edge and center rules.   Let n = number of edges on the base (corners = 2n; edge pieces = 3n or 2n,top & bottom, + n, #edges to connect the bases;
center pieces = 2, top & bottom, + n, the number of lateral faces). Have students explain or show how $(e - 2)^3$ is the rule for the number of hidden cubes. Basically, the corner and edge piece faces are removed from each side and they are finding the volume of the remaining cube.

The following pages could be used as stations. Students working in small groups could circulate through the stations. Have unit cubes available at each station so students can construct the cubes as needed.

Or

Each group of students could get a different sheet. Have unit cubes available to each group so students can construct the cubes as needed. After completing their tasks, students share out how they approached the task and how the table worksheet helped them.

Or

The tasks could be assigned by ability to generalize concepts.   Have unit cubes available to all students.

Follow - up question:

What answers are the same no matter which task was completed? Why?

- All cubes have 8 corners or 8 cubes with 3 exposed faces.
- All rules were the same, except for the last page which uses a rectangular prism that is not a cube.

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What if you had a 12 x 12 x 12 Rubik's® Cube?

Use the patterns in the table to help you answer the questions below. Show or explain how you got your answer to each of the questions.

1) How many smaller unit cubes would make a 12 x 12 x 12 Rubik's Cube?

2) How many unit cubes would have three colored faces showing?

3) How many unit cubes would have only two colored faces exposed?

4) How many unit cubes would have only one face exposed?

5) How many unit cubes would have absolutely no exposed faces?

6) If you would try to build a 12 x 12 x 12 Rubik's Cube, using 3 x 3 x 3 Rubik's Cubes instead of unit cubes, how many would you need?

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Deconstructing Prisms

Teacher Notes: The Rubik's® Cube shown is not the size under consideration.

What if you had a 12 x 12 x 12 Rubik's® Cube?

Use the patterns in the table to help you answer the questions below. Show or explain how you got your answer to each of the questions.

1) How many smaller unit cubes would make a 12 x 12 x 12 Rubik's Cube?

   12 • 12 • 12 = 1,728 cubic units

2) How many unit cubes would have three colored faces showing?

   2•4

   8 cubes which are the corners

3) How many unit cubes would have only two colored faces exposed?

   12 (e-2)

   12 (12-2)

   12(10)

   120 cubes which are on the edges

4) How many unit cubes would have only one face exposed?

   $6(e - 2)^2$

   $6(12 - 2)^2$

   $6(10)^2$

   600 cubes are in the center of each face, including the bases

5) How many unit cubes would have absolutely no exposed faces?

   $(e - 2)^3$

   $(12 - 2)^3$

   10^3

   1000 cubes will be in the interior of the large cube

6) If you would try to build a 12 x 12 x 12 Rubik's Cube, using 3 x 3 x 3 Rubik's Cubes instead of unit cubes, how many would you need?

   Each 3 x 3 x 3 cube has a volume of 3 • 3 • 3 or 27 cubic units.

   1728 ÷ 27 = 64 cubes

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Deconstructing Prisms

In a particular Rubik's® Cube, 1000 unit cubes are hidden unit cubes. Use the patterns in the table to help you answer the questions below. Show or explain how you got your answer to each of the questions.

1) What are the dimensions of this Rubik's Cube?

2) How many unit cubes have only one colored face showing?

3) How many unit cubes have only two colored faces exposed?

4) How many unit cubes have three colored faces exposed?

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Deconstructing Prisms

Teacher Notes: The Rubik's® Cube shown is not the size under consideration.

In a particular Rubik's® Cube, 1000 unit cubes are hidden unit cubes. Use the patterns in the table to help you answer the questions below. Show or explain how you got your answer to each of the questions.

1) What are the dimensions of this Rubik's Cube?

$(e -2)^3 = 1000$

$(e -2)^3 = 10 \bullet 10 \bullet 10$ — 3 factors that equal 1000.

$e - 2 = 10$ — Each factor must equal 10.

$e = 12$ — Each factor is 2 less than the number of unit cubes on one edge.

Older students may have more formal equation solving notation.

2) How many unit cubes have only one colored face showing?

$6(e - 2)^2$

$6(12 -2)^2$

$6(10)^2$

600 cubes are in the center of each face, including the bases

3) How many unit cubes have only two colored faces exposed?

$12 (e-2)$

$12 (12-2)$

$12(10)$

120 cubes which are on the edges

4) How many unit cubes have three colored faces exposed?

$2 \bullet 4$

8 cubes which are the corners

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There is a Rubik’s® Cube of unknown size.
It has 864 unit cubes with only one face showing. Show or explain how you got your answers to the following questions.

1) What are the dimensions of this Rubik’s Cube?

2) How many unit cubes have no faces showing?

3) How many unit cubes have only two colored faces showing?

4) How many unit cubes have three colored faces exposed?

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Deconstructing Prisms

Teacher Notes: The Rubik's® Cube shown is not the size under consideration.

There is a Rubik's® Cube of unknown size. It has 864 unit cubes with only one face showing. Show or explain how you got your answers to the following questions.

1) What are the dimensions of this Rubik's Cube?

$6(e - 2)^2 = 864$
$6(e - 2)^2 = 6\ (144)$ — What times 6 = 864?
$6(e - 2)^2 = 6\ (12 \bullet 12)$ — What number times itself is 144?
e - 2 = 12 — Each factor is 2 less than the edge length.
e = 14 units on each edge

Older students may have more formal equation solving notation.

2) How many unit cubes have no faces showing?

$(e - 2)^3$
$(14 - 2)^3$
$(12)^3$
1728 cubic units are hidden

3) How many unit cubes have only two colored faces showing?

12 (e-2)
12 (14-2)
12(12)
144 cubes are on the edges

4) How many unit cubes have three colored faces exposed?

2•4
8 cubes which are the corners

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Deconstructing Prisms

There is another Rubik's® Cube of unknown size. It has 132 unit cubes with only two faces exposed. Show or explain how you got your answers to the following questions.

1) What are the dimensions of this Rubik's Cube?

2) How many unit cubes have no exposed faces?

3) How many unit cubes have only one face exposed?

4) How many unit cubes have three faces exposed?

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Teacher Notes: The Rubik's® Cube shown is not the size under consideration.

There is another Rubik's® Cube of unknown size. It has 132 unit cubes with only two faces exposed. Show or explain how you got your answers to the following questions.

1) What are the dimensions of this Rubik's Cube?

$12(e-2) = 132$

$12(e-2) = 12(11)$ — What number times 12 = 132?

$e - 2 = 11$ — The factor is 2 less than the edge length.

$e = 13$ units on each edge

2) How many unit cubes have no exposed faces?

$(e-2)^3$

$(13-2)^3$

11^3

1331 cubic units are in the interior of the larger cube

3) How many unit cubes have only one face exposed?

$6(e-2)^2$

$6(13-2)^2$

$6(11)^2$

$6(121)$

726 unit cubes are on the edges

4) How many unit cubes have three faces exposed?

$2 \bullet 4$

8 cubes which are the corners

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You have a Rubik's® Cube with edges of length ***e***. Show or explain how you got your answers to the following questions.

1) If the Rubik's Cube consists of 125 unit cubes, what is the value of ***e***?

2) If the Rubik's Cube has 343 unit cubes that have no faces exposed, what is the value of ***e***?

3) If the Rubik's Cube has 120 unit cubes that have only two faces showing, what is the value of ***e***?

4) If the Rubik's Cube has 486 unit cubes that have only one exposed face, what is the value of ***e***?

5) If the Rubik's Cube has 8 unit cubes with 3 exposed faces, what is the value of **e**?

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Deconstructing Prisms

Teacher Notes: The Rubik's® Cube shown is not the size under consideration.

You have a Rubik's® Cube with edges of length ***e***. Show or explain how you got your answers to the following questions.

1) If the Rubik's Cube consists of 125 unit cubes, what is the value of **e**?

$125 = e^3$
$125 = 5 \bullet 5 \bullet 5$
e = 5 cubes on each edge

2) If the Rubik's Cube has 343 unit cubes that have no faces exposed, what is the value of **e**?

$(e - 2)^3 = 343$
$(e - 2)^3 = 7 \bullet 7 \bullet 7$ What number used as a factor 3 times has a product of 343?
$e - 2 = 7$ Each factor is 2 less than the edge length.
e = 9 units on each edge

3) If the Rubik's Cube has 120 unit cubes that have only two faces showing, what is the value of **e**?

$12 (e - 2) = 120$
$12 (e - 2) = 12 (10)$ What number times 12 = 120?
$e - 2 = 10$ Each factor is 2 less than the edge length.
e = 12 units on each edge

4) If the Rubik's Cube has 486 unit cubes that have only one exposed face, what is the value of **e**?

$6(e - 2)^2 = 486$
$6(e - 2)^2 = 6 \bullet 81$ What number times 6 = 486?
$6(e - 2)^2 = 6 \bullet 9 \bullet 9$ What number times itself = 81?
$e - 2 = 9$ Each factor is 2 less than the edge length.
e = 11 units on each edge

5) If the Rubik's Cube has 8 unit cubes with 3 exposed faces, what is the value of **e**?

All Rubik's Cubes have 8 corners with 3 exposed faces because the base of the cubes is a square. There is no way to determine a single value for **e**.

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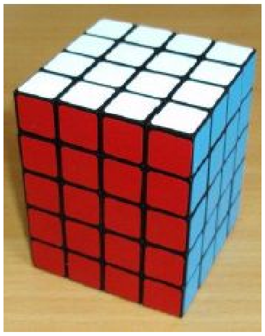

This puzzle pictured is not your typical Rubik's® Cube because it is not a cube.

1) Why is this puzzle not a cube?

2) What is a geometric term we could use for this puzzle?

3) What are its dimensions?

4) How many unit cubes does the puzzle contain? Show or explain how you got your answer.

5) How many of the unit cubes have three faces showing?

6) How many of the unit cubes have exactly two faces exposed?

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7) How many of the unit cubes have only one exposed face?

8) How many of the unit cubes have no exposed faces?

Challenge: Write the rules for rectangular prisms of any size using variables. How many variables will you need? Define each of the variables.

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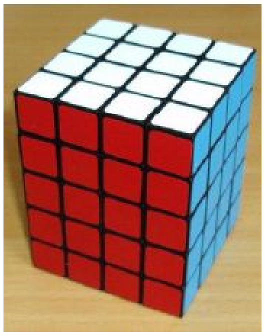
Teacher Notes:



This puzzle pictured is not your typical Rubik's Cube because it is not a cube.

1) Why is this puzzle not a cube?
All of the faces are not squares. The bases are squares, but the lateral faces are rectangles. All the faces of a cube are squares.

2) What is a geometric term we could use for this puzzle?
Rectangular prism, regular rectangular prism
(The bases are squares, regular polygons.)

3) What are its dimensions?
4 x 4 x 5

4) How many unit cubes does the puzzle contain? Show or explain how you got your answer.
4 x 4 x 5 = 80 cubic units

5) How many of the unit cubes have only three faces showing? Show or explain how you got your answer.
8 All rectangular prisms have 8 corners with 3 exposed faces because the bases are rectangles.

6) How many of the unit cubes have exactly two faces exposed? Show or explain how you got your answer.
28. Each base has e-2 or 2 edge pieces on each side. That makes 8 edge pieces per base for a total of 16 edge pieces. The edges connecting the bases have a length of 5. There are e - 2 or 3 edge pieces on each of these edges for a total of 12 edges pieces. 16 + 12 = 28 edge pieces with only two exposed faces in the prism.

7) How many of the unit cubes have only one exposed face?
The cubes with one exposed face are in the center of each face of the prism. For the bases, that would be $2(e-2)^2$. $2(4-2)^2 = 2(2)^2$ or 8 cubes on the bases. Each lateral face would have (x - 2)(y - 2) cubes with 1 exposed face. So, for the 4 lateral faces there would be 4(4 -2)(5 -2) = 4 • 2 • 3 or 24 cubes with 1 exposed face. 24 + 8 = 32 unit cubes with one exposed face.

8) How many of the unit cubes have no exposed faces?
The hidden prism inside has dimensions of 3 x 2 x 2. Therefore, there are 12 unit cubes with no exposed faces.

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Teacher Notes:

Challenge: Write the rules for rectangular prisms of any size using variables. How many variables will you need? Define each of the variables.

Since prisms are 3 dimensional, there might be three variables. Some traditional choices are l, w, h or x, y, z. For this example, **e** represents the length of the base edge and **h** represents the height of the prism.

Volume: e • e • h
3 exposed faces: 2 • e
2 exposed faces: 2 • 4(e -2) + 4(h-2)
1 exposed face: 2 • $(e - 2)^2$ + 4(h-2)(e-2)
0 exposed faces: (h-2)(e-2)

Some students might be challenged to write the rules for any rectangular prism using 3 variables.

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