Using patterns, students write expressions that determine the number of unit cubes with any given number of exposed faces. They will test their generalizations in situations where they are given one variable (a prism with \(x\) units with 1 exposed face) and must determine all other characteristics.

**TEKS Math Concepts & Standards**

**Math 6.9 (A, C) Expressions, equations, and relationships.** The student applies mathematical process standards to use equations and inequalities to represent situations. The student is expected to: (A) write one-variable, one-step equations and inequalities to represent constraints or conditions within problems;

**Math 5.4 (B) Algebraic reasoning.** The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to: (B) represent and solve multi-step problems involving the four operations with whole numbers using equations with a letter standing for the unknown quantity;

**Math 4.5 (A) Algebraic reasoning.** The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to: (A) represent multi-step problems involving the four operations with whole numbers using strip diagrams and equations with a letter standing for the unknown quantity;

**Math 6.8 (A-D) Expressions, equations, and relationships.** The student applies mathematical process standards to use geometry to represent relationships and solve problems. The student is expected to: (A) extend previous knowledge of triangles and their properties to include the sum of angles of a triangle, the relationship between the lengths of sides and measures of angles in a triangle, and determining when three lengths form a triangle; (B) model area formulas for parallelograms, trapezoids, and triangles by decomposing and rearranging parts of these shapes; (C) write equations that represent problems related to the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers; and (D) determine solutions for problems involving the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers.

**Materials**

- A Rubik’s Mini, Rubik’s Cube and Rubik’s Master (one set for each student, group, or learning station)
  - Sets may be borrowed for 6 weeks from [www.youcandothecube.com/lending-library/](http://www.youcandothecube.com/lending-library/)
- 27 unit cubes per student, group, or learning station
- Handouts for each student, group, or learning station

**Background Vocabulary:**

- prism, regular rectangular prism
- bases
- faces, lateral faces
- unit cube
- edge
- corner (vertex, vertices)
- exposed

**Teacher Pages**

- With answer keys and additional information follow student pages
Rubik's Mini Vocabulary

• The mathematical term for the Rubik's Mini shown is **prism**. A prism has two **bases** and rectangular **faces**.

The top and bottom of a prism are called the **bases**. What shape are the bases of this prism?

The sides of a prism are called **lateral faces**, or sometimes just **faces**. The faces of a prism are always parallelograms. In right prisms, the faces are rectangles. Why?

How many lateral faces are there?

What determines the number of lateral faces in a prism?

If the bases of a prism are regular polygons, the prism is called **regular**. The Rubik's Mini is a regular rectangular prism. Explain why.

• The size of a prism is determined by the number of smaller cubes or **unit cubes** on the **edge** of the prism. An edge is a line segment where exactly 2 faces meet. How many edges does a Rubik's Mini have?

What is the length of the edge on a Rubik's Mini?

• A **corner** or **vertex** is the point where exactly 3 faces meet. How many corners or vertices does a Rubik's Mini have?

• Are there any unit cubes in the interior of a Rubik's Mini that can’t be seen?
Deconstructing Prisms

Teacher Notes: This page could be classwork after some general instruction to prisms. It reinforces the prism vocabulary using a Rubik's Mini. It could also be used as the instruction for prism vocabulary.

Rubik's Mini Vocabulary

- The mathematical term for the Rubik's® Mini shown is **prism**. A prism has two **bases** and rectangular **faces**.

The top and bottom of a prism are called the **bases**. What shape are the bases of this prism? **Square**

The sides of a prism are called **lateral faces**, or sometimes just **faces**. The faces of a prism are always parallelograms. In right prisms, the faces are rectangles. Why? It is assumed here that students are working with right prisms. Perhaps the student's working definition of a prism at this point is that the bases are parallel and segments connecting the vertices are perpendicular to the bases. Therefore, the lateral faces would have 4 right angles making them rectangles. It is important that students know the term **lateral faces** because there are times when the term faces will include the bases. This can be confusing even when using context clues. Using the term lateral faces helps to clarify.

How many lateral faces are there? **4** If this is being used for instruction, the teacher may also ask, "How many lateral faces there would be if the bases were triangular or some other polygon?".

What determines the number of lateral faces in a prism? Students should generalize that the number of lateral faces is equal to the number of sides on the base.

If the bases of a prism are regular polygons, the prism is called **regular**. The Rubik's Mini is a regular rectangular prism. Explain why. The base of a Rubik's Mini is a square. A square is a regular polygon because all the sides and angles are congruent. A square is a rectangle. A regular rectangular prism is also called a square prism. If all the faces, including the bases, are squares, the prism is a cube.

- The size of a prism is determined by the number of smaller cubes or **unit cubes** on the **edge** of the prism. An edge is a line segment where exactly 2 faces meet. How many edges does a Rubik's Mini have? **12** Students may generalize that the number of lateral faces is equal to the number of sides of the base times 3.

What is the length of the edge on a Rubik's Mini? **2 units** The teacher may want to reinforce the difference between linear units (units of perimeter), square units (units of area), and cubic units (units of volume). An extension could be made to exponents and the vocabulary of squaring and cubing a number.

- A **corner** or **vertex** is the point where exactly 3 faces meet. How many corners or vertices does a Rubik's Mini have? **8** Students may generalize that the number of vertices is equal to the number of sides of the base times 2.

- Are there any unit cubes in the interior of a Rubik's Mini that can't be seen? **No**. Students could use smaller unit cubes to prove this. This sets up the work that follows which focuses on the number of exposed faces of the unit cubes which make up the larger cube.
Deconstructing Prisms

- Look for patterns in the number of faces that are exposed on the unit cubes. You will be counting the colored sides of each of the unit cubes.
- To help you write the rule, you may want to look at what you learned on the previous page.

<table>
<thead>
<tr>
<th>Prism</th>
<th>Edge length of the prism</th>
<th>Number of cubic units</th>
<th>3 faces exposed</th>
<th>2 faces exposed</th>
<th>1 face exposed</th>
<th>0 faces exposed</th>
</tr>
</thead>
</table>

| Rules: Is there a way to predict the number of cubes and faces based on the size of the edge of a cube? |
|-------|---------|---------|---------|---------|---------|
|       | Hint: How many squares on 1 edge? | (corners) | (edges) | (center) | Hint: How many cubes can't be seen? |
### Deconstructing Prisms

#### Teacher Notes

Students may work individually or in pairs to complete the chart. Remind students that they are focusing on the faces of the unit cubes. This will be important as they may confuse **edge of the prism** with **edge pieces** which have 2 exposed faces. The teacher may want to have unit cubes available so that students can build the first two or three cubes to see how faces of the unit cubes become “hidden” in the larger cube. Have students look for generalizations after completing each larger cube so they are working towards the rule throughout the task.

- Look for patterns in the number of faces that are exposed on the unit cubes. You will be counting the colored sides of each of the unit cubes.
- To help you write the rule, you may want to look at what you learned on the previous page.

<table>
<thead>
<tr>
<th>Prism</th>
<th>Edge length of the prism</th>
<th>Number of cubic units</th>
<th>3 faces exposed</th>
<th>2 faces exposed</th>
<th>1 face exposed</th>
<th>0 faces exposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>8</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2·2·2</td>
<td>4 · 2</td>
<td>12(2-2)</td>
<td>6(2-2)^2</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2^3</td>
<td></td>
<td>12(0)</td>
<td>6(0)^2</td>
<td>(2-2)^3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>27</td>
<td>8</td>
<td>12</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3·3·3</td>
<td>4 · 2</td>
<td>12(3-2)</td>
<td>6(3-2)^2</td>
<td>(3-2)^3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3^3</td>
<td></td>
<td>12(1)</td>
<td>6(1)^2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>64</td>
<td>8</td>
<td>24</td>
<td>24</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4·4·4</td>
<td>4 · 2</td>
<td>12(4-2)</td>
<td>6(4-2)^2</td>
<td>(4-2)^3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4^3</td>
<td></td>
<td>12(2)</td>
<td>6(2)^2</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>125</td>
<td>8</td>
<td>54</td>
<td>54</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5·5·5</td>
<td>4 · 2</td>
<td>12(5-2)</td>
<td>6(5-2)^2</td>
<td>(5-2)^3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5^3</td>
<td></td>
<td>12(3)</td>
<td>6(3)^2</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>216</td>
<td>8</td>
<td>96</td>
<td>96</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6·6·6</td>
<td>4 · 2</td>
<td>12(6-2)</td>
<td>6(6-2)^2</td>
<td>(6-2)^3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6^3</td>
<td></td>
<td>12(4)</td>
<td>6(4)^2</td>
<td>4</td>
</tr>
</tbody>
</table>

#### Rules: Is there a way to predict the number of cubes and faces based on the size of the edge of a cube?

- **Edge**
  - **Corner**
    - **Number of cubes on 1 edge?**
      - e = e · e · e
    - **(corner)**
      - 8
    - **(edge pieces)**
      - 12(e - 2) or 2·4(e-2)+4(e-2) or 3·4(e-2)
    - **(center pieces)**
      - 6(e-2)^2 or 2·4(e-2)^2
  - **Hint:** How many cubes can’t be seen? (e - 2)^3
Have students share their rules. Have them explain how they formulated the rules so that they recognize equivalent expressions such as those for edge pieces.

Challenge: On the vocabulary page, students may have generalized about the number of corners and edges in a prism. Have students articulate why the 2, 12, and 6 remain constant in the corner, edge and center rules. Let \( n \) = number of edges on the base (corners = 2\( n \); edge pieces = 3\( n \) or 2\( n \), top & bottom, + \( n \), #edges to connect the bases; center pieces = 2, top & bottom, + \( n \), the number of lateral faces). Have students explain or show how \((e - 2)^3\) is the rule for the number of hidden cubes. Basically, the corner and edge piece faces are removed from each side and they are finding the volume of the remaining cube.

The following pages could be used as stations. Students working in small groups could circulate through the stations. Have unit cubes available at each station so students can construct the cubes as needed.

Or

Each group of students could get a different sheet. Have unit cubes available to each group so students can construct the cubes as needed. After completing their tasks, students share out how they approached the task and how the table worksheet helped them.

Or

The tasks could be assigned by ability to generalize concepts. Have unit cubes available to all students.

Follow-up question:

What answers are the same no matter which task was completed? Why?

- All cubes have 8 corners or 8 cubes with 3 exposed faces.
- All rules were the same, except for the last page which uses a rectangular prism that is not a cube.
What if you had a 12 x 12 x 12 Rubik’s Cube?

Use the patterns in the table to help you answer the questions below. Show or explain how you got your answer to each of the questions.

1) How many smaller unit cubes would make a 12 x 12 x 12 Rubik’s Cube?

2) How many unit cubes would have three colored faces showing?

3) How many unit cubes would have only two colored faces exposed?

4) How many unit cubes would have only one face exposed?

5) How many unit cubes would have absolutely no exposed faces?

6) If you would try to build a 12 x 12 x 12 Rubik’s Cube, using 3 x 3 x 3 Rubik’s Cubes instead of unit cubes, how many would you need?
What if you had a 12 x 12 x 12 Rubik’s Cube?

Use the patterns in the table to help you answer the questions below. Show or explain how you got your answer to each of the questions.

1) How many smaller unit cubes would make a 12 x 12 x 12 Rubik’s Cube?
   \[ 12 \cdot 12 \cdot 12 = 1,728 \text{ cubic units} \]

2) How many unit cubes would have three colored faces showing?
   \[ 2 \cdot 4 \]
   8 cubes which are the corners

3) How many unit cubes would have only two colored faces exposed?
   \[ 12 \cdot (12-2) \]
   120 cubes
   edges

4) How many unit cubes would have only one face exposed?
   \[ 6(e - 2)^2 \]
   \[ 6(12 - 2)^2 \]
   \[ 6(10)^2 \]
   600 cubes are in the center of each face, including the bases

5) How many unit cubes would have absolutely no exposed faces?
   \[ (e - 2)^3 \]
   \[ (12 - 2)^3 \]
   \[ 10^3 \]
   1000 cubes will be in the interior of the large cube

6) If you would try to build a 12 x 12 x 12 Rubik’s Cube, using 3 x 3 x 3 Rubik’s Cubes instead of unit cubes, how many would you need?
   Each 3 x 3 x 3 cube has a volume of 3 \cdot 3 \cdot 3 or 27 cubic units.
   \[ 1728 \div 27 = 64 \text{ cubes} \]
Deconstructing Prisms

In a particular Rubik’s Cube, 1000 unit cubes are hidden unit cubes. Use the patterns in the table to help you answer the questions below. Show or explain how you got your answer to each of the questions.

1) What are the dimensions of this Rubik’s Cube?

2) How many unit cubes have only one colored face showing?

3) How many unit cubes have only two colored faces exposed?

4) How many unit cubes have three colored faces exposed?
In a particular Rubik's Cube, 1000 unit cubes are hidden unit cubes. Use the patterns in the table to help you answer the questions below. Show or explain how you got your answer to each of the questions.

1) What are the dimensions of this Rubik's Cube?

\[(e - 2)^3 = 1000\]
\[(e - 2)^3 = 10 \cdot 10 \cdot 10\] 3 factors that equal 1000.
\[e - 2 = 10\] Each factor must equal 10.
\[e = 12\] Each factor is 2 less than the number of unit cubes on one edge.

Older students may have more formal equation solving notation.

2) How many unit cubes have only one colored face showing?

\[6(e - 2)^2\]
\[6(12 - 2)^2\]
\[6(10)^2\]

600 cubes are in the center of each face, including the bases.

3) How many unit cubes have only two colored faces exposed?

\[12(e - 2)\]
\[12(12 - 2)\]
\[12(10)\]

120 cubes which are on the edges.

4) How many unit cubes have three colored faces exposed?

\[2 \cdot 4\]

8 cubes which are the corners.
Deconstructing Prisms

There is a Rubik's Cube of unknown size. It has 864 unit cubes with only one face showing. Show or explain how you got your answers to the following questions.

1) What are the dimensions of this Rubik's Cube?

2) How many unit cubes have no faces showing?

3) How many unit cubes have only two colored faces showing?

4) How many unit cubes have three colored faces exposed?
There is a Rubik's Cube of unknown size. It has 864 unit cubes with only one face showing. Show or explain how you got your answers to the following questions.

1) What are the dimensions of this Rubik's Cube?

\[
6(e - 2)^2 = 864 \\
6(e - 2)^2 = 6 (144) \quad \text{What times 6 = 864?} \\
6(e - 2)^2 = 6 (12 \cdot 12) \quad \text{What number times itself is 144?} \\
e - 2 = 12 \quad \text{Each factor is 2 less than the edge length.} \\
e = 14 \text{ units on each edge}
\]

Older students may have more formal equation solving notation.

2) How many unit cubes have no faces showing?

\[
(e - 2)^3 \\
(14 - 2)^3 \\
(12)^3 \\
1728 \text{ cubic units are hidden}
\]

3) How many unit cubes have only two colored faces showing?

\[
12 (e - 2) \\
12 (14 - 2) \\
12(12) \\
144 \text{ cubes are on the edges}
\]

4) How many unit cubes have three colored faces exposed?

\[
2 \cdot 4 \\
8 \text{ cubes which are the corners}
\]
There is another Rubik’s Cube of unknown size. It has 132 unit cubes with only two faces exposed. Show or explain how you got your answers to the following questions.

1) What are the dimensions of this Rubik’s Cube?

2) How many unit cubes have no exposed faces?

3) How many unit cubes have only one face exposed?

4) How many unit cubes have three faces exposed?
There is another Rubik's Cube of unknown size. It has 132 unit cubes with only two faces exposed. Show or explain how you got your answers to the following questions.

1) What are the dimensions of this Rubik's Cube?

\[ 12 \cdot (e-2) = 132 \]
\[ 12 \cdot (e-2) = 12 \cdot (11) \]
\[ e - 2 = 11 \]
The factor is 2 less than the edge length.
\[ e = 13 \text{ units on each edge} \]

2) How many unit cubes have no exposed faces?

\[ (e - 2)^3 \]
\[ (13 - 2)^3 \]
\[ 11^3 \]
1331 cubic units are in the interior of the larger cube

3) How many unit cubes have only one face exposed?

\[ 6(e - 2)^2 \]
\[ 6(13 - 2)^2 \]
\[ 6(11)^2 \]
\[ 6(121) \]
726 unit cubes are on the edges

4) How many unit cubes have three faces exposed?

\[ 2 \cdot 4 \]
8 cubes which are the corners
Deconstructing Prisms

You have a Rubik’s Cube with edges of length $e$. Show or explain how you got your answers to the following questions.

1) If the Rubik’s Cube consists of 125 unit cubes, what is the value of $e$?

2) If the Rubik’s Cube has 343 unit cubes that have no faces exposed, what is the value of $e$?

3) If the Rubik’s Cube has 120 unit cubes that have only two faces showing, what is the value of $e$?

4) If the Rubik’s Cube has 486 unit cubes that have only one exposed face, what is the value of $e$?

5) If the Rubik’s Cube has 8 unit cubes with 3 exposed faces, what is the value of $e$?
You have a Rubik’s® Cube with edges of length $e$.
Show or explain how you got your answers to the following questions.

1) If the Rubik’s Cube consists of 125 unit cubes, what is the value of $e$?

\[
125 = e^3
\]
\[
125 = 5 \cdot 5 \cdot 5
\]
$e = 5$ cubes on each edge

2) If the Rubik’s Cube has 343 unit cubes that have no faces exposed, what is the value of $e$?

\[
(e - 2)^3 = 343
\]
\[
(e - 2)^3 = 7 \cdot 7 \cdot 7
\]
Each factor is 2 less than the edge length.
$e - 2 = 7$
$e = 9$ units on each edge

3) If the Rubik’s Cube has 120 unit cubes that have only two faces showing, what is the value of $e$?

\[
12(e - 2) = 120
\]
\[
12(e - 2) = 12 \cdot 10
\]
Each factor is 2 less than the edge length.
$e - 2 = 10$
$e = 12$ units on each edge

4) If the Rubik’s Cube has 486 unit cubes that have only one exposed face, what is the value of $e$?

\[
6(e - 2)^2 = 486
\]
\[
6(e - 2)^2 = 6 \cdot 81
\]
Each factor is 2 less than the edge length.
$e - 2 = 9$
$e = 11$ units on each edge

5) If the Rubik’s Cube has 8 unit cubes with 3 exposed faces, what is the value of $e$?

All Rubik’s Cubes have 8 corners with 3 exposed faces because the base of the cubes is a square. There is no way to determine a single value for $e$. 

www.YouCanDoTheCube.com

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