

Geometry of Solids



National Standards

Instructional programs from pre-kindergarten through grade 12 should enable all students to:

Analyze properties and determine attributes of two- and three-dimensional objects

- Visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections
- Use geometric models to gain insights into, and answer questions in, other areas of mathematics
- Apply transformations and use symmetry to analyze mathematical situations

Common Core State Standards

CCSS.MATH.CONTENT.7.G.B.6

Solve real-world and mathematical problems involving angle measure, area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Standards for Mathematical Practice

- MP.1. Make sense of problems and persevere in solving them.
- MP.2. Reason abstractly and quantitatively.
- MP.3. Construct viable arguments and critique the reasoning of others.
- MP.4. Model with mathematics.
- MP.5. Use appropriate tools strategically.
- MP.6. Attend to precision.
- MP.7. Look for and make use of structure.
- MP.8. Look for and express regularity in repeated reasoning.

21st Century Skills

Learning and Innovation Skills

- Critical Thinking and Problem Solving
 - Exercising sound reasoning in understanding
- Life and Career Skills
 - Initiative & Self-Direction
 - Monitoring one's own understanding and learning needs

Materials

- Class set of Rubik's Cubes
- Pages 1 and 2 (Stage 1: GET TO KNOW YOUR RUBIK'S CUBE) of Solution Guide for each student
- Page 3 (Stage 2: SOLVE THE WHITE CROSS) of Solution Guide for each student
- Homework Activity ONE
- Pencils and Paper

Objective

1. Introduce the Rubik's Cube and become familiar with its structure and associated terminology in
2. order for students to understand the solution guide and better understand solid geometry.
3. Students will
 - Know the terminology associated with cubes and other geometric solids
 - Understand the relationships between the center, edge, and corner pieces of the Rubik's Cube
 - Learn the language of the solution guide and how to solve the white cross in order to complete the first step in solving the cube.
4. Students will
 - Be able to solve the white cross
 - Know two methods/algorithms

Notes to Teacher

- There are videos on YouTube showing how to solve the Rubik's Cube. This one uses the same
- Solution Guide your students have: <https://goo.gl/gBFbLp>
- If students have trouble solving the white cross after reading the solution guide, explain the general strategy is to put the white edge pieces down in the bottom layer, turn the bottom layer until the piece lines up with its correct center mate, and then spin that layer 180 degrees to put the white edge piece on the top.
- This is an excellent lesson to get the students to better understand rotations. All the corner and edge pieces rotate around the center pieces (center of rotation).

Procedure

1. Introduce the Rubik's Cube by giving a brief history. (5 min.)

Erno Rubik's (an architectural design instructor born in Budapest, Hungary) initial attraction to inventing the Cube was not in producing the best selling toy puzzle in history. The structural design problem interested Rubik; he asked, "How could the blocks move independently without falling apart?" In Rubik's Cube, twenty-six individual little cubes or "cubies" make up the big Cube. Each layer of nine cubies can twist and the layers can overlap. Any three squares in a row, except diagonally, can join a new layer. Rubik's initial attempt to use elastic bands failed, his solution was to have the blocks hold themselves together by their shape. Rubik hand carved and assembled the little cubies together. He marked each side of the big Cube with adhesive paper of a different color, and started twisting. The Cube, as a puzzle, was invented in the spring of 1974, when the twenty-nine year old Rubik discovered it was not so easy to realign the colors to match on all six sides. He was not sure he would ever be able to return his invention to its original position. He theorized that by randomly twisting the Cube he would never be able to fix it in a lifetime, which later turns out to be more than correct. He began working out a solution, starting with aligning the eight corner cubies. He discovered certain sequences of moves for rearranging just a few cubies at a time. Within a month, he had the puzzle solved and an amazing journey lay ahead.
2. Distribute cubes and pages 1 and 2 (stage 1) of the solution guide. Read through both pages with students. Be sure to explain the counter-clockwise/inverted language. (15 min.)
3. Discuss with students: (5 min.)
 - A geometric solid is a 3-dimensional object with faces, vertices and edges.
 - A cube is a prism with 6 faces, all congruent squares.
 - The Rubik's Cube is known as a 3x3. If it were made up entirely of smaller cubes, it would have a volume of 27 units and a surface area of 54 units.
 - The Rubik's Cube has 8 corner pieces, 12 edge pieces and 6 (fixed) center pieces.
 - Ask the students: since $8 + 12 + 6 = 26$, what happened to the 27th piece?
4. Distribute page 3 (Stage 2) of the solution guide. Read through this stage with the students and demonstrate the method and algorithm. (15 min.)
5. Pass out worksheet and review. (5 min.)

Geometry of Solids

Name: _____

Date: _____

Vocabulary:

faces

edges

vertices

prism

bases (top and bottom)

lateral faces

lateral edges

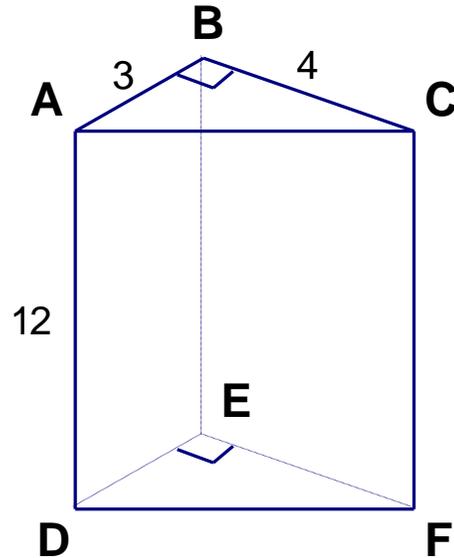
cube

Exercises

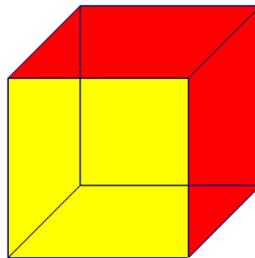
1. Name the bases of the prism: $\triangle ABC$ and
2. Name the lateral faces of the prism.

3. What shape are the lateral faces of the prism?
4. Name the lateral edges of the prism.

5. What is the height of the prism?
6. What is the perimeter of the base?
7. What is the total area of all of the faces?
8. How many vertices does the figure have?
9. How many edges does the figure have?
10. How many faces does the figure have?



Assume the cube below has edge length 4cm.



11. How many faces does a cube have?

12. What is the shape of each face on a cube?
13. What is the area of one of the faces of a cube with edge lengths of 4cm?
14. How many edges does a cube have?
15. How many vertices does a cube have?
16. If the cube above was a Rubik's Cube, it would have how many:



- a. Center pieces?
- b. Corner pieces?
- c. Edge pieces?

The **BASE AREA** of a prism is defined as the combined area of both bases. *Commonly, it is referred to as Top & Bottom. For the Rubik's Cube, we call them Up and Down faces.*

17. What is the base area of the cube above?

The **LATERAL AREA** of a prism is defined as the combined area of all the lateral faces. *Commonly, it is referred to as Front, Back, Right and Left. It is the same for the Rubik's Cube.*

18. What is the lateral area of the cube above?
19. What is the total surface area of the cube above?

Geometry of Solids

Vocabulary:

faces **flat surfaces of a three-dimensional figure**

edges **a line segment that joins two vertices on the boundary or where faces meet.**

vertices **points on a 3D figure where three faces intersect, or point where two or more straight lines meet**

prism **a polyhedron consisting of two parallel, congruent faces called bases**

bases (top and bottom)

the bottom of a plane figure or three-dimensional figure

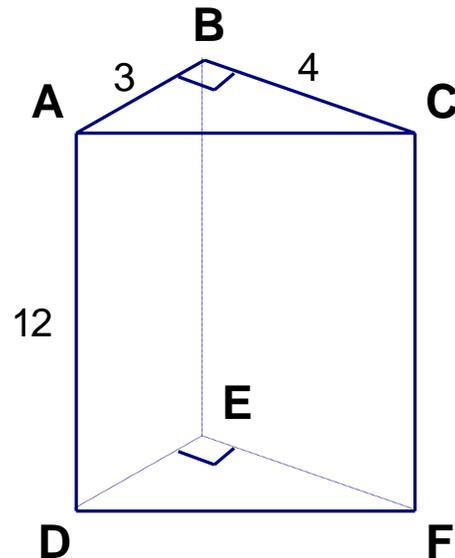
lateral faces **the faces that join the bases of a solid**

lateral edges **the edges that form the lateral faces of a solid**

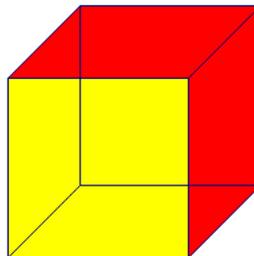
cube **a solid figure with six square faces**

Exercises

1. Name the bases of the prism: $\triangle ABC$ and $\triangle DEF$
2. Name the lateral faces of the prism.
ABED BCFE ACFD
3. What shape are the lateral faces of the prism?
Rectangles
4. Name the lateral edges of the prism.
AD BE CF
5. What is the height of the prism? **12 units**
6. What is the perimeter of the base? **12 units**
7. What is the total area of all of the faces? **144 units²**
8. How many vertices does the figure have? **6**
9. How many edges does the figure have? **9**
10. How many faces does the figure have? **5**



Assume the cube below has edge length 4 cm.



11. How many faces does a cube have? **6**

12. What is the shape of each face on a cube? **square**
13. What is the area of one of the faces of a cube with edge lengths of 4cm? **16 cm²**
14. How many edges does a cube have? **12**
15. How many vertices does a cube have? **8**
16. If the cube above was a Rubik's Cube, it would have how many:
-  a. Center pieces? **6**
- b. Corner pieces? **8**
- c. Edge pieces? **12**

The BASE AREA of a prism is defined as the combined area of both bases. *Commonly, it is referred to as Top & Bottom. For the Rubik's Cube, we call them Up and Down faces.*

17. What is the base area of the cube above? **32 cm²**

The LATERAL AREA of a prism is defined as the combined area of all the lateral faces. *Commonly, it is referred to as Front, Back, Right and Left. It is the same for the Rubik's Cube.*

18. What is the lateral area of the cube above? **64 cm²**
19. What is the total surface area of the cube above? **96 cm²**