Middle School

Geometry of Solids

National Standards

Instructional programs from pre-kindergarten through grade 12 should enable all students to:

Analyze properties and determine attributes of two- and three-dimensional objects

- Visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections
- Use geometric models to gain insights into, and answer questions in, other areas of mathematics
- Apply transformations and use symmetry to analyze mathematical situations

Texas Essential Knowledge & Skills

Mathematics 8.7B Expressions, equations, and relationships.

The student applies mathematical process standards to use geometry to solve problems

use previous knowledge of surface area to make connections to the formulas for lateral and total surface area and determine solutions for problems involving rectangular prisms, triangular prisms, and cylinders;

Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding

(A) apply mathematics to problems arising in everyday life, society, and the workplace
(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
(E) create and use representations to organize, record, and communicate mathematical ideas;
(F) analyze mathematical relationships to connect and communicate mathematical ideas; and
(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

21st Century Skills

Learning and Innovation Skills

- Critical Thinking and Problem Solving
  - Exercising sound reasoning in understanding
- Life and Career Skills
  - Initiative & Self-Direction
    - Monitoring one's own understanding and learning needs

Materials

- Class set of Rubik's Cubes
- Pages 1 and 2 (Stage 1: GET TO KNOW YOUR RUBIK'S CUBE) of Solution Guide for each student
- Page 3 (Stage 2: SOLVE THE WHITE CROSS) of Solution Guide for each student
- Homework Activity ONE
- Pencils and Paper

Objective

- Introduce the Rubik's Cube and become familiar with its structure and associated terminology in
- order for students to understand the solution guide and better understand solid geometry.
- Students will
  - Know the terminology associated with cubes and other geometric solids
  - Understand the relationships between the center, edge, and corner pieces of the Rubik's Cube
  - Learn the language of the solution guide and how to solve the white cross in order to complete the first step in solving the cube.
- Students will
  - Be able to solve the white cross
  - Know two methods/algorithms

Notes to Teacher

- There are videos on YouTube showing how to solve the Rubik's Cube. This one uses the same Solution Guide your students have: https://goo.gl/gBFbLp
- If students have trouble solving the white cross after reading the solution guide, explain the general strategy is to put the white edge pieces down in the bottom layer, turn the bottom layer until the piece lines up with its correct center mate, and then spin that layer 180 degrees to put the white edge piece on the top.
- This is an excellent lesson to get the students to better understand rotations. All the corner and edge pieces rotate around the center pieces (center of rotation).

Procedure

1. Introduce the Rubik's cube by giving a brief history. (5 min.)

   Erno Rubik's (an architectural design instructor born in Budapest, Hungary) initial attraction to inventing the Cube was not in producing the best selling toy puzzle in history. The structural design problem interested Rubik; he asked, "How could the blocks move independently without falling apart?" In Rubik's Cube, twenty-six individual little cubes or "cubies" make up the big Cube. Each layer of nine cubies can twist and the layers can overlap. Any three squares in a row, except diagonally, can join a new layer. Rubik's initial attempt to use elastic bands failed, his solution was to have the blocks hold themselves together by their shape. Rubik hand carved and assembled the little cubies together. He marked each side of the big Cube with adhesive paper of a different color, and started twisting. The Cube, as a puzzle, was invented in the spring of 1974, when the twenty-nine year old Rubik discovered it was not so easy to realign the colors to match on all six sides. He was not sure he would ever be able to return his invention to its original position. He theorized that by randomly twisting the Cube he would never be able to fix it in a lifetime, which later turns out to be more than correct. He began working out a solution, starting with aligning the eight corner cubies. He discovered certain sequences of moves for rearranging just a few cubies at a time. Within a month, he had the puzzle solved and an amazing journey lay ahead.

2. Distribute cubes and pages 1 and 2 (stage 1) of the solution guide. Read through both pages with students. Be sure to explain the counter-clockwise/inverted language. (15 min.)

3. Discuss with students: (5 min.)
   - A geometric solid is a 3-dimensional object with faces, vertices and edges.
   - A cube is a prism with 6 faces, all congruent squares.
   - The Rubik's Cube is known as a 3x3. If it were made up entirely of smaller cubes, it would have a volume of 27 units and a surface area of 54 units.
   - The Rubik's Cube has 8 corner pieces, 12 edge pieces and 6 (fixed) center pieces.
   - Ask the students: since 8 + 12 + 6 = 26, what happened to the 27th piece?

4. Distribute page 3 (Stage 2) of the solution guide. Read through this stage with the students and demonstrate the method and algorithm. (15 min.)

5. Pass out worksheet and review. (5 min.)

Geometry of Solids

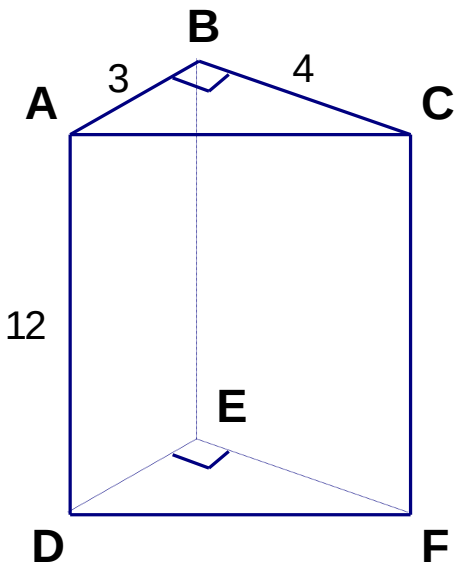
Name:_____
Date:_____

Vocabulary:

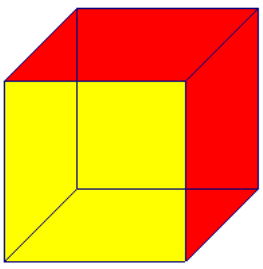
faces	bases (top and bottom)
edges	lateral faces
vertices	lateral edges
prism	cube

Exercises

1. Name the bases of the prism: *∆ABC and* _____
2. Name the lateral faces of the prism.
   _____ _____ _____
3. What shape are the lateral faces of the prism?
4. Name the lateral edges of the prism.
   _____ _____ _____
5. What is the height of the prism?
6. What is the perimeter of the base?
7. What is the total area of all of the faces?
8. How many vertices does the figure have?
9. How many edges does the figure have?
10. How many faces does the figure have?



Assume the cube below has edge length 4cm.



11. How many faces does a cube have?

12. What is the shape of each face on a cube?

13. What is the area of one of the faces of a cube with edge lengths of 4cm?

14. How many edges does a cube have?

15. How many vertices does a cube have?

16. If the cube above was a Rubik's Cube, it would have how many:

a. Center pieces?

b. Corner pieces?

c. Edge pieces?

The BASE AREA of a prism is defined as the combined area of both bases. *Commonly, it is referred to as Top & Bottom. For the Rubik's Cube, we call them Up and Down faces.*

17. What is the base area of the cube above?

The LATERAL AREA of a prism is defined as the combined area of all the lateral faces. *Commonly, it is referred to as Front, Back, Right and Left. It is the same for the Rubik's Cube.*

18. What is the lateral area of the cube above?

19. What is the total surface area of the cube above?

Geometry of Solids

Vocabulary:

faces flat surfaces of a three-dimensional figure

edges a line segment that joins two vertices on the boundary or where faces meet.

vertices points on a 3D figure where three faces intersect, or point where two or more straight lines meet

prism a polyhedron consisting of two parallel, congruent faces called bases

bases (top and bottom) the bottom of a plane figure or three-dimensional figure

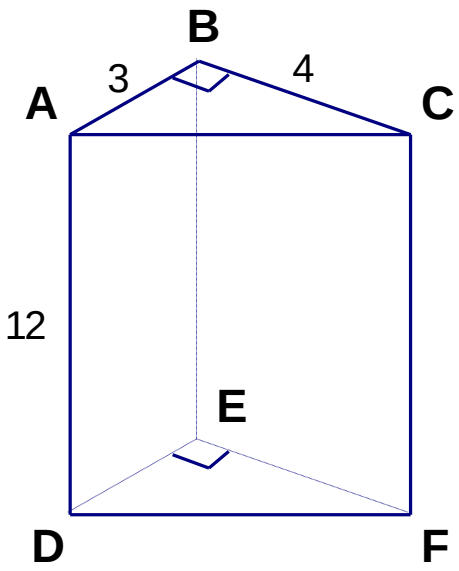
lateral faces the faces that join the bases of a solid

lateral edges the edges that form the lateral faces of a solid

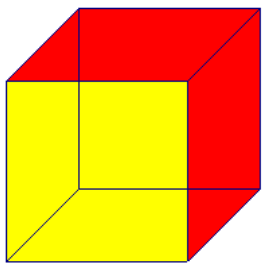
cube a solid figure with six square faces

Exercises

1. Name the bases of the prism: ΔABC and ΔDEF
2. Name the lateral faces of the prism.
   ABED BCFE ACFD
3. What shape are the lateral faces of the prism?
   Rectangles
4. Name the lateral edges of the prism.
   AD BE CF
5. What is the height of the prism? 12 units
6. What is the perimeter of the base? 12 units
7. What is the total area of all of the faces? 144 units^2
8. How many vertices does the figure have? 6
9. How many edges does the figure have? 9
10. How many faces does the figure have? 5



Assume the cube below has edge length 4 cm.



11. How many faces does a cube have? 6

12. What is the shape of each face on a cube? square

13. What is the area of one of the faces of a cube with edge lengths of 4cm? 16 cm^2

14. How many edges does a cube have? 12

15. How many vertices does a cube have? 8

16. If the cube above was a Rubik's Cube, it would have how many:

a. Center pieces? 6

b. Corner pieces? 8

c. Edge pieces? 12

The BASE AREA of a prism is defined as the combined area of both bases. *Commonly, it is referred to as Top & Bottom. For the Rubik's Cube, we call them Up and Down faces.*

17. What is the base area of the cube above? 32 cm^2

The LATERAL AREA of a prism is defined as the combined area of all the lateral faces. *Commonly, it is referred to as Front, Back, Right and Left. It is the same for the Rubik's Cube.*

18. What is the lateral area of the cube above? 64 cm^2

19. What is the total surface area of the cube above? 96 cm^2