



Lesson 1 Introductory Geometry: Measurement



National Standards

Instructional programs for Geometry grades 5th and 6th should enable all students to:

- understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects
- Select and apply techniques and tools to accurately find length, area, [and] volume ...to appropriate levels of precision
- Identify, compare, and analyze attributes of two- and three dimensional shapes and develop vocabulary to describe the attributes

21st Century Skills

Learning and Innovation Skills

Critical Thinking and Problem Solving

- Exercising sound reasoning in understanding
- Understanding the interconnections among systems
- Identifying and asking significant questions that clarify various points of view and lead to better solutions
- Framing, analyzing and synthesizing information in order to solve problems and answer questions

Creativity and Innovation

- Acting on creative ideas to make a tangible and useful contribution to the domain in which the innovation occurs

Life and Career Skills

Initiative & Self-Direction

- Defining, prioritizing and completing tasks without direct oversight
- Utilizing time efficiently and managing workload

Leadership & Responsibility

Using interpersonal and problem-solving skills to influence and guide others toward a goal

**Texas
Essential
Knowledge
& Skills
(TEKS)**

Math 1.7 (C) Geometry and measurement

The student applies mathematical process standards to select and use units to describe length and time. The student is expected to:

- (C) measure the same object/distance with units of two different lengths and describe how and why the measurements differ;

Math 2.9 (F) Geometry and measurement.

The student applies mathematical process standards to select and use units to describe length, area, and time. The student is expected to:

- (F) use concrete models of square units to find the area of a rectangle by covering it with no gaps or overlaps, counting to find the total number of square units, and describing the measurement using a number and the unit;

Mathematical Process Standards (K-12)

- (A) apply mathematics to problems arising in everyday life, society, and the workplace;
- (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
- (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
- (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
- (E) create and use representations to organize, record, and communicate mathematical ideas;
- (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
- (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Students will be introduced to, understand, and practice the following concepts:

Objective

- Width & Length
- Perimeter of a square = $4 \times \text{Side Length}$
- Perimeter of a rectangle = $(2 \times \text{Length}) + (2 \times \text{Width})$
- Area of a square or rectangle = $\text{Length} \times \text{Width}$
- The importance of using a standard and predefined unit of measure
- Also, students will be introduced to the Rubik's Cube as well as to the meaning and relativity of the term "cubie" as a unit of measure for Rubik's Cube puzzles.

Materials

- Class set of Rubik's Cubes (one per student)
- Day 1 Worksheet (one per student)—(see end of file)
- Day 1 Worksheet (KEY)—(see end of file)
- Class set of rulers (inches)
- Photos (or online access to photos) of various Rubik's Cube-related puzzles (see website for picture link under Step 3.1 below).

Procedure

Step 1— Introduction

1. Inform the class that for the next four days they will be learning math skills while having fun playing with a famous geometric toy.
2. Hold up a Rubik's Cube for the whole class to see.
3. Ask students if they have ever seen, heard of, or even played with a Rubik's Cube before.
4. Solicit responses.
5. Ask those who have played with a Rubik's Cube if they have ever solved it themselves or seen someone solve it in person. (Some students might tell stories of how their older sibling can solve it if they remove the stickers or take it apart and put it back together in order. Some students may actually know a friend or relative who can solve it.)
6. Pass out the class set of Rubik's Cubes AFTER first explaining that you do not want the students to twist them or scramble them up. Kids will be very tempted to play with them. Aside from just feeling them, holding them, and looking them over, strongly encourage the class at this point to just keep them "solved" and "unscrambled."
7. Explain to the class that over the next four days they will be learning and practicing many geometric concepts, all with their Rubik's Cubes.
8. Explain that today they will be learning about length, width, perimeter, and area of a two-dimensional shape.

Procedure

Step 2— Instruction

1. Ask each student to put their Rubik's Cube on their desk with their favorite color side face-up towards them.
2. Ask students what they believe the horizontal (left-right) width of their favorite side is.
3. Solicit responses. (Correct answers may vary: Some students may say three cubes wide. Others may approximate two inches wide or 5 to 6 centimeters wide. Some will want to know the exact width of the Rubik's Cube, so you can let them know that it's pretty close to 2 inches, or 5 centimeters, wide.)
4. Explain to the class the importance of agreeing upon a standard unit for measurement: Without an agreed-upon standard unit of measurement, we can often confuse one another with misleading information. For example, many famous scientific and costly mistakes have actually been made by experts who accidentally used the wrong unit of measurement (inches/feet instead of centimeters/meters), such as when the famous Hubble Space Telescope was first sent to outer space—its lens was wrongly shaped because its designer had accidentally used the wrong unit of measure! A careless mistake that cost the scientific community MILLIONS of dollars!
5. Explain that for the sake of simplicity (and fun!), the class will be measuring their Rubik's Cubes today in terms of "cubies." A "cubie" is the width of just one of the 26 little cubes that make up a Rubik's Cube. Therefore, a Rubik's Cube is three cubies wide.
6. Let the class quickly verify this themselves.
7. Ask the class what the length of the top side is, in cubies (i.e., the distance from the front of the cube to the back of the cube). (Answer: 3 cubies)
8. Establish with the class that both the width and the length of the top side of the Rubik's Cube are both 3 cubies and that the top side's dimensions are therefore 3 cubies by 3 cubies (3 x 3).
9. Ask what you call a shape that has the same width and length as well as four right (square) angles in its corners. (Answer: a square)

Procedure

10. Explain to the class that each of the six sides of the Rubik's Cube is a square that is 3 cubies wide and 3 cubies long.
11. Let the class quickly verify this for all six sides.
12. Introduce the concept of perimeter: Perimeter is, literally, the measure ("-meter") around ("peri-") a shape, such as a square or rectangle. So, perimeter is the measured distance around a shape.
13. Ask the class, "If you were a little ant starting in the very corner of the top side of your cube, how many cubie lengths would you have to travel to walk around the perimeter of the top side?"
14. Solicit Responses (Answer: $3 + 3 + 3 + 3 = 12$ cubies)
15. Explain that to find the perimeter of a geometric shape, you just add up the length of each side.
16. Explain that for a square, finding the perimeter is easy because you can just find one side (3 cubies in this case) and multiply by four (3×4) to find the perimeter (12 cubies), since all four sides of a square are always the same.
17. Introduce the concept of area: the total two-dimensional (flat) space taken up by a two-dimensional shape, such as a circle, triangle, rectangle, or in this case, a square.
18. Ask the class how many flat, colored tiles are found on the top side of their cube. (Answer: 9 tiles) (Older Rubik's Cubes may have stickers, rather than tiles.)
19. Explain to the class that these tiles are like square units, or in this case, square cubies. Therefore, the area of the top side of their cube is 9 square cubies, or 9 cubies².
20. Explain that the answer to an area problem is always stated in "square units." In our case, we will use the unit of measurement "square cubies."
21. Explain that using the Rubik's Cube makes finding the square units of a shape easy: just count the little squares. This can also be done on graph paper or grids on which

Procedure

22. Explain that using the Rubik's Cube makes finding the square units of a shape easy: just count the little squares. This can also be done on graph paper or grids on which shapes are laid out with square units inside of them clearly visible for counting. However, this is rarely ever the case in real life (e.g., on a tile floor). Therefore, mathematicians have come up with a more useful (and faster!) way for finding the area of a square or rectangle: length x width (area formula)
23. Present the formula for determining the area of a square or rectangle: Area = Length x Width
24. Ask the class to use the area formula to calculate the area of the top side of their Rubik's Cube. (Answer: Area = $3 \times 3 = 9$ square cubies = 9 cubies^2).
25. Reason together with the class that it indeed makes sense that the area of the top side of their cubes is 9 square cubies because it literally has 9 square tiles on that side, as confirmed before.
26. Now pass out the Day 1 Worksheet and confirm with the students the meaning and accuracy of the first row of information: the width, length, perimeter, and area of one side of a Rubik's Cube.
27. Then confirm with the students that in actual inches the Rubik's Cube can be measured according to the same methods to determine its perimeter of 9 inches and square area of (approximately) 5 square inches. (Refer to the rest of the first row of the handout.)

Procedure

Step 3—Guided Practice

1. Now show the class other kinds of Rubik's Cubes that have been made, namely $2 \times 2 \times 2$, $4 \times 4 \times 4$, and $5 \times 5 \times 5$ Rubik's Cubes, etc. *Use the internet to find pictures of various sizes of Rubik's Cubes to project for the class to see, or print out the pictures to share with the class. (Can find images at <http://rubiks.com/store/cubes/>)*
2. Share the names of each of the cubes:
 - a. Pocket Cube = $2 \times 2 \times 2$
 - b. Rubik's Cube (the original) = $3 \times 3 \times 3$
 - c. Rubik's Revenge = $4 \times 4 \times 4$
 - d. Professor's Cube = $5 \times 5 \times 5$
3. Tell the class that they will fill in their worksheet to calculate both the *perimeter* and *area* of just one side of each of these variously sized cubes.
4. Remind the students to use the shortcuts already discussed, such as four times the length of one side to find the *perimeter* of a square, and Width x Length for determining *area*.
5. Students may work in partners to calculate the *perimeter* and *area* of one side of each of the variously sized cubes.

6. Check answers together. (See Day 1 Worksheet Key for Answers.)
7. You may have to clarify for the class that the size of a cubie is actually relative to the toy cube it is a part of. For example, the cubies for a Pocket Cube are much larger than the cubies for a 5x5x5.
8. Clarify for the students that the area formula works just as well for a rectangle as it does for a square. Just multiply the width and length of the rectangle for its total area.
9. Also clarify that the perimeter of a rectangle is NOT found with the same shortcut as that for a square. Instead, for a rectangle's perimeter, multiply the width twice and the length twice, and then add the two products: Perimeter of a Rectangle = (2 x Length) + (2 x Width)

Procedure

Step 4— Independent Practice

1. Pass out the class set of rulers (inches) and assign to the students the following measurement tasks with answers recorded on the Day 1 Worksheet. Students may work alone or together, but sharing cubes, laid end-to-end, will ultimately result in quicker and more accurate answers. Allow students to explore the room as necessary to finish the following tasks (All answers vary according to each individual object in the classroom):
 - a. Measure, both in *cubies* and *inches*, the area of their student desk.
 - b. Measure both in *cubies* and *inches*, the area of a piece of notebook paper.
 - c. Measure, both in *cubies* and *inches*, the area of their student textbook cover.
 - d. Measure both in *cubies* and *inches*, the area of any one white board/blackboard in the classroom.
 - e. Measure, both in *cubies* and *inches*, the area of a window, door, or other large rectangle in the room.
 - f. Advanced: Estimate, both in *cubies* and *inches*, the area of the sole of their shoe. (Note: this will be an estimate with no readily available precise answer.)
 - g. Advanced: Estimate, both in *cubies* and *inches*, the area of any irregular shape of the student's choice. (Note: this will be an estimate with no readily available precise answer.)
2. Remind students that as they may lay Rubik's Cubes side-by-side to make measurements, that each cube is three cubies long.

3. After a few minutes, reconvene and share and discuss answers together.
4. Discuss any patterns, observations, or ideas discovered by the students.
5. Discuss, explain, and reason through any unexpected differences in answers shared among the students.
6. Review any possible reasoning mistakes that were made in the measuring process and discuss how to avoid such mistakes in the future.
7. Clarify any other misunderstandings and/or answer any final questions.
1. Review the main concepts discussed today: *Width*, *Length*, *Perimeter* (and its formulas), *Area* (and its formula), the importance of using a predefined standard unit of measure, the meaning and relativity of the term “*cubie*” as a unit of measure, and any other strategies discovered & shared by the students through this learning process.

Procedure

Step 5— Closure & Review