Introductory Geometry 3: Perimeter & Area of Composite Figures

Note to Teacher: This is the third lesson in a series of four geometry lessons. Previous lessons are referred within as Day 1, Day 2...and so on. You can find the other lessons at You Can Do The Rubik’s Cube website.

National Standards
NCTM

Instructional programs for Geometry grades 5th and 6th should enable all students to:

- understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects
- Select and apply techniques and tools to accurately find length, area, [and] volume ...to appropriate levels of precision
- Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes

Texas Essential Knowledge & Skills (TEKS)

Math 4.5D Algebraic reasoning.
The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:
solve problems related to perimeter and area of rectangles where dimensions are whole numbers.

Mathematical Process Standards K-12
The student uses mathematical processes to acquire and demonstrate mathematical understanding.

(A) to apply mathematics to problems arising in everyday life, society, and the workplace

(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;

(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;

(E) create and use representations to organize, record, and communicate mathematical ideas;

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

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21st Century Skills

Learning and Innovation Skills

Critical Thinking and Problem Solving
- Exercising sound reasoning in understanding
- Understanding the interconnections among systems
- Identifying and asking significant questions that clarify various points of view and lead to better solutions
- Framing, analyzing and synthesizing information in order to solve problems and answer questions

Creativity and Innovation
- Developing, implementing and communicating new ideas to others

Life and Career Skills

Initiative & Self-Direction
- Defining, prioritizing and completing tasks without direct oversight
- Utilizing time efficiently and managing workload

Leadership & Responsibility
- Using interpersonal and problem-solving skills to influence and guide others toward a goal

Objective

Students will be introduced to, understand, and practice the following concepts:
- Perimeter of composite figures
- Area of composite figures
Material

1. Class set of Rubik’s Cubes (one per student or pair of students)
2. Way to visually display to the class each of the composite figures demonstrated & discussed in this lesson as shown.
3. Day 2 Worksheet (KEY)—(see Day 2 lesson plans)
4. Day 3 Worksheet (one for each student—see below)
5. Day 3 Homework Sheet (one for each student—see below) Day 3 Homework Sheet (KEY)
6. Optional: Colored pencils, crayons, or markers (Red, Blue, Orange, and Green)

Students bring:

7. Completed Day 2 Worksheet (one for each student)—(see Day 2 lesson plans)
8. Rubik’s Cube
9. Solution Guide

Procedure

Introduction

1. Take out homework and discuss answers from the homework portion of the Day 2 Worksheet sheet, sharing discoveries, observations, challenges, etc.
2. Take out Rubik’s Cubes (most likely, several students were able to use their solution guides to complete Stages 2, 3, and possibly more)
3. Inform the class that today they will become more familiar with their Rubik’s Cubes while learning about how to determine both the perimeter and area of composite figures

Instruction

1. Review the definition of area: the total two-dimensional (flat) space taken up by a two-dimensional shape, such as a circle, triangle, rectangle, or in today’s case, a composite figure.
2. Present to the class figure 1 from the attached figure page
3. Ask how students can determine the perimeter and area of this strange polygon (solicit suggestions from the class).
4. Point out that the above polygon is actually composed of three conjoined rectangles, as shown in figure 2 on the attached figure page
5. Explain that by breaking down the unfamiliar shape (first figure) into familiar shapes (second figure), it becomes very easy to determine both the perimeter and area of unusual shapes.
6. Remind the class of the definition of perimeter from Day 1:
   a. perimeter: the measure (“-meter”) around (“peri-”) a shape, such as a square or rectangle. So, perimeter measures the total distance around a shape
7. Explain to the class that to determine the perimeter of the above figure, we just add up the lengths of all of its exterior (outside) sides, as shown figure 3 on the attached figure page
8. Explain that using the unit grid as a guide, the length of each side can be determined, as shown above. Also explain that opposite congruent sides are of the same length (for example, the top left side is also 5 units long just like the lower left side).

9. Using the above information, the perimeter of the composite figure above is, proceeding clockwise from the left side:
\[3 + 5 + 1 + 2 + 1 + 2 + 3 + 2 + 2 + 2 + 5 = 30\text{ units}.\]

10. Check for understanding.

11. Next, ask the class how they can determine the total area of the above composite figure. (If necessary, remind them to break it down into its composite rectangles A, B & C as shown in the second figure above.)

12. Demonstrate to the class that by simply calculating the individual areas using the area formula for a rectangle and adding up the results, the total area of the composite figure can be determined.

13. Demonstrate calculating the area for rectangles A, B, and C using the area formula:
   a. Rectangle A = \(L \times W = 2 \times 1 = 2\text{ units}^2\)
   b. Rectangle B = \(L \times W = 9 \times 3 = 27\text{ units}^2\)
   c. Rectangle C = \(L \times W = 2 \times 2 = 4\text{ units}^2\)
   d. Total Area = \(2 + 27 + 4 = 33\text{ units}^2\)

14. You may need to explain that the actual width of Rectangle B is a bit tricky to determine because its full width must be considered, not just its labeled segments. Therefore, its width is \(5 + 2 + 2 = 9\text{ units wide}.\)

15. Check for understanding.
1. Explain that the class will now practice determining the perimeter and area of composite figures by using their Rubik’s Cubes.

2. Ask the class to take out their Rubik’s Cubes and Solution Guides, and, if not already done, tell them to complete through Solve Layer One, Step 2: the White Cross.

3. Make sure that every student has the White Cross completed and no more than that done.

4. You may need to ask those students who are already more proficient than others to help make sure their neighbors are correctly and completely done with their White Crosses.

5. Display the following diagram of the White Cross to the class and ask the class to determine both its perimeter and total area in centimeters and centimeters$^2$ respectively, using the given diagram that one square is approximately 2 centimeters in length: as shown figure 4 on the attached figure page.

6. Guide students as needed in determining the total perimeter of this composite figure: Perimeter = $2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 24$ centimeters.

7. Guide students as needed in determining the total area of the composite figure by breaking down the figure into 3 recognizable rectangles as shown below:

8. Then, guide students as needed in finding the total of each rectangle as follows:
   a. Rectangle A= Length x Width= 2 cm x 2 cm= 4 cm$^2$
   b. Rectangle B = Length x Width= 6 cm x 2 cm= 12 cm$^2$
   c. Rectangle C = Length x Width = 2 cm x 2 cm = 4 cm$^2$
   d. Total Area of the White Cross= $4 + 12 + 4 = 20$ cm$^2$

9. Check for understanding
1. Now ask the class to correctly complete Layer One, Step 3-Solve the White Corners, on their Rubik's Cubes, making sure that the edge pieces of the white face correctly match the center piece of each adjacent side (red edges with red center, blue edges with blue center, etc.)

2. Have students check their neighbors' cubes to make sure everyone is done with Layer One, Step 3 and no further. Assist as needed.

3. Pass out the Day 3 Worksheet to each student.

4. Remind the class that each face is determined by its center square. For example, the White Face is determined by the white center square.

5. Ask the class to find the Red Face on their Rubik's Cube while still keeping the White Face on top.

6. Explain to the class that everyone's Red Face will probably look different from that of other students. However, at least the top three squares should be red, matching the center square. All other squares may or may not be red.

7. Explain that you are not right now concerned if the other red squares, if any, on the Red Face are actually in the correct spot, so long as the top three squares are in fact red, thus matching the center red square.

8. Next, instruct the class to make the first figure on the Day 3 Worksheet match their own Red Face on their Rubik's Cubes coloring (or shading) in the figure with their pencils- shaded portions should only be those matching the center square of the Red Face.

9. (Optional: Students may want to use red colored pencils or crayons for this project. If you choose to use color, make sure to pass out the appropriate supplies at this point.)

10. After making their diagram match the Red Face of their Rubik's Cube, ask the class to use their understanding of composite figures to determine both the perimeter and area of their own Red Face and then to write their totals in the space provided, labeling answers in cm or cm². Again, students should use the estimate that each square is approximately 2 cm long.

11. Tell the students to repeat these instructions for the Blue, Orange, and Green faces as well.

12. Monitor and assist students as needed. You may also want to assign "expert" peer helpers to assist.

13. After a few minutes, ask students to share their findings, discoveries, observations, challenges, and questions.
**Teacher Notes**

1. Tell students to turn to pages 10-11 of their Solution Guides—Solve the Middle Layer.

2. Briefly review these three pages with the students, making sure that they still know how to hold their cubes and how to rotate the faces of their Rubik's Cube according to the pictures provided.

3. Spend time in class helping (and with students helping other students) the class review pages 1-9 of the cube. Put your "expert cubers" to work who have shown a natural ability to understand the solution guide and communicate it to others. Discourage these helpers from "solving" their peers' cubes for them. Instead, encourage these helpers to explain and demonstrate to their neighbors by using their own cubes.

4. In the last five minutes of class, pass out the Day 3 Homework Sheet to each student and explain the homework assignment to the class:
   
a. For Figures 1 & 2 of the homework, students are to determine the area and perimeter of each of the two composite figures. Encourage students to label the length of each separate side, as well as to break down each composite figure into simple rectangles that can be easily calculated for their area.

   b. For Figures 3 & 4 of the homework, students are to create, draw, and shade-in their own composite figures, as well as determine each figure's perimeter and area in units and square units.

   c. Complete the middle layer of their Rubik's Cube and bring their Rubik's Cube AND Solution Guide with them to class the next day. Encourage students that it's OK if they can't solve the Middle Layer by tomorrow. They will have more time after the unit to complete this goal. The important thing is that they are struggling and trying hard to grasp the Solution Guide instructions.

5. Check for understanding of the homework assignment.
Appendix I: Figures

Figure 1: Procedure - Instruction step 2

Figure 2: Procedure - Instruction Step 4
Figure 3: Procedure - Instruction Step 7

Figure 4: Procedure - Guided Practice-Step 5

Figure 5: Procedure - Guided Practice-Step 7
Lesson 3
Introductory Geometry: Area of Composite Figures

Worksheet

Red Face

Perimeter of Red Figure = cm
Area of Red Figure = cm²

Blue Face

Perimeter of Blue Figure = cm
Area of Blue Figure = cm²

Orange Face

Perimeter of Orange Figure = cm
Area of Orange Figure = cm²

Green Face

Perimeter of Green Figure = cm
Area of Green Figure = cm²

Challenge: Total Area of Squares located on their correct face, (including the White Face) = cm²
Lesson 3
Introductory Geometry: Area of Composite Figures

Name __________________________ Date ____________________

Homework

<table>
<thead>
<tr>
<th>Figure 1</th>
<th>Figure 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Figure 1 Image" /></td>
<td><img src="image2.png" alt="Figure 2 Image" /></td>
</tr>
<tr>
<td>Figure 1 Perimeter = _______ units</td>
<td>Figure 2 Perimeter = _______ units</td>
</tr>
<tr>
<td>Figure 1 Area = _______ units²</td>
<td>Figure 2 Area = _______ units²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Figure 3 (Make your own!)</th>
<th>Figure 4 (Make your own!)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3.png" alt="Figure 3 Image" /></td>
<td><img src="image4.png" alt="Figure 4 Image" /></td>
</tr>
<tr>
<td>Figure 3 Perimeter = _______ units</td>
<td></td>
</tr>
<tr>
<td>Figure 3 Area = _______ units²</td>
<td></td>
</tr>
<tr>
<td>Figure 4 Perimeter = _______ unit:</td>
<td></td>
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<tr>
<td>Figure 4 Area = _______ units²</td>
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</tbody>
</table>
Lesson 3
Introductory Geometry: Area of Composite Figures

Name __________________________ Date ____________________

Homework

Answer Key

Figure 1

Figure 1 Perimeter = 80 units
Figure 1 Area = 138 units²

Figure 2

Figure 2 Perimeter = 68 units
Figure 2 Area = 116 units²

Figure 3 (Make your own!)

Figure 3 Perimeter = _______ units
Figure 3 Area = _______ units²

Figure 4 (Make your own!)

Figure 4 Perimeter = _______ units
Figure 4 Area = _______ units²