Introduction to Geometry 2:
Identify Geometric Attributes & Volume

Note to Teacher: This is the second lesson in a series of four geometry lessons. Previous lessons are referred within as Day 1, Day 2...and so on. You can find the other lessons at www.YouCanDoTheCube.com

National Standards
NCTM

Instructional programs for Geometry grades 5th and 6th should enable all students to:
- understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects
- Select and apply techniques and tools to accurately find length, area, [and] volume ...to appropriate levels of precision

Common Core State Standards

CCSS.MATH.CONTENT.5.MD.C.5.B
Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.

CCSS.MATH.CONTENT.7.G.B.6
Solve real-world and mathematical problems involving angle measure, area, surface area, and volume.

Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.
MP.2. Reason abstractly and quantitatively.
MP.3. Construct viable arguments and critique the reasoning of others.
MP.5. Use appropriate tools strategically.
MP.6. Attend to precision.
MP.7. Look for and make use of structure.
MP.8. Look for and express regularity in repeated reasoning.

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21st Century Skills

Learning and Innovation Skills

Critical Thinking and Problem Solving

- Exercising sound reasoning in understanding
- Understanding the interconnections among systems
- Identifying and asking significant questions that clarify various points of view and lead to better solutions
- Framing, analyzing and synthesizing information in order to solve problems and answer questions

Creativity and Innovation

- Developing, implementing and communicating new ideas to others

Life and Career Skills

Initiative & Self-Direction

- Defining, prioritizing and completing tasks without direct oversight
- Utilizing time efficiently and managing workload

Leadership & Responsibility

- Using interpersonal and problem-solving skills to influence and guide others toward a goal

Objectives

Students will be introduced to, understand, and practice the following concepts:

- Polygon and Polyhedron
- Face(s), Edge(s), and Vertex (vertices)
- Prism and Cube
- Length, Width, Height
- Volume of a Rectangular Prism and Cube
Materials

1. Class set of Rubik’s Cubes (one per student or pair of students)
2. Class set of rulers (inches)
4. Several samples of real-life rectangular prisms of various sizes & dimensions, preferably enough for each table group to share one.
   - Examples: empty tissue boxes, cereal boxes, other food and/or storage cardboard boxes,
   - Starting with the letter “A”, clearly label each prism on the outside with a large marker so that each prism is labeled with a different letter (A, B, C, D….etc.).
   - You will want to save these boxes for the Day 4 lesson as well.
5. Day 1 Worksheet (KEY)
   see [Day 1 lesson plans](#)
6. Day 2 Worksheet (one for each student)
7. Day 2 Worksheet (KEY)
   Teacher must provide the answers for each prism (Prism A, B, C, etc.) based on the exact prisms the teacher brings for the class to study.
8. Photos (or online access to photos) of various Rubik’s Cube-related puzzles

Students bring:

9. Completed Day 1 Worksheet (one for each student)
10. Rubik’s Cube

Procedure

Introduction

1. Take out homework and discuss answers from the Day 1 worksheet, sharing discoveries, observations, challenges, etc.
2. Ask students to take out their Rubik’s Cubes (most likely still very mixed up)
3. Inform the class that today they will become more familiar with their Rubik’s Cubes while learning about polygons, polyhedrons, faces, edges, vertices, length, width, height, rectangular prisms, cubes and volume.

Note to Teacher: At this point, the class will be very excited to discover that there actually is a solution guide that explains how to solve the Rubik’s Cube.
1. Turn together to page 1 of the Solution Guide and read aloud and discuss the section -- Get to Know Your Rubik’s Cube.

2. After reading and discussing pages 1-3, and while holding their cubes in their hands, use the Rubik’s Cube to explain and demonstrate to the class the meanings of *polygon, polyhedron, face, edge, vertex, length, width, height, prism,* and *cube* using the following definitions for each (demonstrate each definition on the Rubik’s Cube, and then ask each student to *feel* the definitions for themselves on their own Rubik’s Cubes):

   a. *Polygon:* a two-dimensional shape made up of three or more straight lines (line segments).—Note the square polygon represented by one face of a Rubik’s Cube.

   b. *Polyhedron:* a three-dimensional object with flat surfaces made up of polygons—Note that a Rubik’s Cube is a polyhedron.

   c. *Face:* a flat surface of a polyhedron—Note the faces of a Rubik’s Cube.

   d. *Edge:* where two faces of a polyhedron come together—Note the edges of a Rubik’s Cube.

   e. *Vertex* (pl. *vertices*): A point where three or more edges come together—Note the vertices on a Rubik’s Cube.

   f. *Length, width, and height:* the three dimensions of a three-dimensional geometric shape—Note the three dimensions of length, width, and height on a Rubik’s Cube.

   g. *Prism:* a polyhedron made of two parallel and congruent faces that are polygons AND all remaining faces are rectangles—Note that a Rubik’s Cube is a prism.

   h. *Cube:* A special prism in which all 6 faces are congruent square faces—Note that a Rubik’s Cube is a cube.

3. Ask the following questions while telling the class to feel, point at, and confirm for themselves their answers based on their own Rubik’s Cubes:

   a. How many *faces* does a Rubik’s Cube have (6 faces)

   b. How many *edges* does a Rubik’s Cube have (12 edges)

   c. How many *vertices* does a Rubik’s Cube have (8 vertices)

   d. In *cubies,* what is the *length* of a Rubik’s Cube (3 cubies)

   e. In *cubies,* what is the *width* of a Rubik’s Cube (3 cubies)

   f. In *cubies,* what is the *height* of a Rubik’s Cube (3 cubies)

   g. What special geometric name can we give to a Rubik’s Cube because its two bases are polygons AND its four other faces are rectangles? (Prism—you may have to remind the class that a square is indeed also a polygon AND a rectangle)

   h. What special geometric name can we name a Rubik’s Cube because it is a prism with 6 congruent, square faces? (Cube)
Procedure

**Instruction**

4. Explain the definition of **volume**: the total cubic units found within a three-dimensional space.

5. Hold up a Rubik’s Cube and ask how many **cubic cubies** (cubies^3) compose the bottom layer of a Rubik’s Cube (9 cubies^3—keeping in mind that we are measuring three-dimensional space, not the actual shapes of the many plastic parts that make up a Rubik’s Cube when you break it apart).

6. Ask how many cubic cubies are in the middle layer of a Rubik’s Cube (9 cubies^3—again, keeping in mind that there actually is no cubie piece in the exact middle of a Rubik’s Cube—we are measuring three-dimensional space, not the number of parts comprising the mechanism of the cube).

7. Ask how many cubic cubies are in the top layer of a Rubik’s Cube (9 cubies^3).

8. Ask the class how many total cubic cubies there are if we sum up all of the cubic cubies from all three layers (9 + 9 + 9 = 27 cubies^3).

9. Explain to the class that volume is the total of cubic units (in this case, cubic cubies) found in a polyhedron.

10. Explain that there is a shortcut formula for determining the total volume of a rectangular prism, such as a Rubik’s Cube:

**Volume of a Rectangular Prism = Length x Width x Height**

11. Explain that, therefore, the volume of a Rubik’s Cube can be quickly determined by multiplying its length (3 cubies) by its width (3 cubies) by its height (3 cubies), thus giving a total volume of 27 cubies^3.

12. Confirm that this total matches the previous total provided in Step 9 when we just added the layers’ volumes together.

(You may want to remind the class that we are multiplying, NOT adding, the length, width, and height. Therefore if a student finds a final answer of 9 cubies^3, then that student probably made the simple mistake of adding instead of multiplying.)

**Procedure**

**Guided Practice**

1. Pass out the Day 2 Worksheet to each student.

2. Now show the class other kinds of Rubik’s Cubes that have been made, namely Rubik’s Mini (2x2), Rubik’s Master (4x4), Professor’s Cube (5x5) (as was shown in class for the Day 1 lesson).

3. Ask the class to determine the faces, edges, vertices, length, width, height and volume of each of the cube shapes, and to record their answers on their Day 2 worksheets in the spaces provided.

4. After a few minutes, check answers with the class using the answer key provided.
Procedure

Independent Practice

1. Show and/or pass around the labeled rectangular prisms (empty boxes) you have brought with you, preferably one for each table group or pair of students (see supplies list above).

2. Pass out an inch ruler to each student.

3. Ask the class to work together using their rulers to determine the number of faces, edges, vertices, length, width, height (all rounded to the nearest inch) and total volume (in cubic inches) of the rectangular prism their group/pair has been assigned.

4. Tell the class to record their data on the Day 2 Worksheet in the same row as the prism’s label (Prism A, Prism B, Prism C, etc.).

5. After a few minutes (preferably at a clearly understood audible signal), each group passes their prisms to the next group so that each group has a new prism to study for determining its number of faces, edges, vertices, length, width, height, and total volume in cubic inches.

6. As they had done on the first prism, students record the data on the worksheet in the corresponding row for that prism (Prism A, B, C, etc.).

7. Continue this pattern until all the prisms have circulated to each group or until only about 10-15 minutes remain in class.

8. Review answers for each prism with the class, beginning with Prism A.

9. Discuss any patterns discovered, interesting observations made, etc.

10. Answer any questions & clarify any misunderstandings about the meaning and measurement of faces, edges, vertices, length, width, height, and volume.

Procedure

Closure & Review

1. Review the definitions and formulas learned today: polygon, polyhedron, face, edge, vertex (vertices), length, width, height, prism, cube, and volume.

2. Explain that once these concepts are well understood for rectangular prisms (especially volume), they are much easier to understand and calculate for triangular prisms, pyramids, cylinders, and other complicated three-dimensional polygons and shapes.

3. Tell students to turn to pages 4-9 of their Solution Guides.
4. Briefly review these pages with the students, making sure that they still know how to hold their cubes and how to rotate the faces of their Rubik’s Cube according to the pictures provided.

5. What’s Next? - Homework/ Assessment/ Enrichment
   a. Complete the Daisy, White Cross and White Corners (pages 4-9 in Solution Guide). Encourage students that it’s OK if they can’t solve Stages 2 & 3 by tomorrow. They will have more time after the unit to complete this goal. The important thing is that they are struggling and trying hard to grasp the Solution Guide instructions.
   b. Bring their Rubik’s Cube AND Solution Guide with them to class the next day.
   c. Find three more rectangular prisms (including one cube) at home and measure them and record their faces, edges, vertices, length, width, height, and volume (both in cubic cubies and cubic inches) on the Day 2 Worksheet in the space provided.
   d. Bring the completed Day 2 Worksheet to the next class day.
Lesson 2
Introductory Geometry Lessons:
Identifying Geometric Attributes / Volume

Name_____________________________Date_____________________

Volume of a Rectangular Prism = Length x Width x Height

<table>
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<tr>
<th>Polyhedron</th>
<th>Faces</th>
<th>Edges</th>
<th>Vertices</th>
<th>Length</th>
<th>Width</th>
<th>Height</th>
<th>Volume (cubies³)</th>
<th>Volume (inches³)</th>
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<tbody>
<tr>
<td>Rubik’s Cube</td>
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Volume of a Rectangular Prism = Length x Width x Height

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<th>Vertices</th>
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<th>Width</th>
<th>Height</th>
<th>Volume (cubies$^3$)</th>
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