Middle School

Introductory Geometry: Surface Area/ Nets

Note to Teacher: *This is the fourth lesson in a series of four geometry lessons. Previous lessons are referred within as Day 1, Day 2…and so on. You can find the other lessons at You Can Do The Rubiks Cube website.*

National Standards
NCTM

Instructional programs for Geometry grades 5th and 6th should enable all students to:

- understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects
- Select and apply techniques and tools to accurately find length, area, [and] volume …to appropriate levels of precision
- Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes

Common Core State Standards

CCSS.MATH.CONTENT.6.G.A 4

Solve real-world and mathematical problems involving area, surface area, and volume

Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Standards for Mathematical Practice

MP.1. Make sense of problems and persevere in solving them.

MP.2. Reason abstractly and quantitatively.

MP.3. Construct viable arguments and critique the reasoning of others.

MP.4. Model with mathematics.

MP.5. Use appropriate tools strategically.

MP.6. Attend to precision.

MP.7. Look for and make use of structure.

MP.8. Look for and express regularity in repeated reasoning.

21st Century Skills

Learning and Innovation Skills

Critical Thinking and Problem Solving

- Exercising sound reasoning in understanding
- Understanding the interconnections among systems
- Identifying and asking significant questions that clarify various points of view and lead to better solutions
- Framing, analyzing and synthesizing information in order to solve problems and answer questions

Creativity and Innovation

- Developing, implementing and communicating new ideas to others

Life and Career Skills

Initiative & Self-Direction

- Defining, prioritizing and completing tasks without direct oversight
- Utilizing time efficiently and managing workload

Leadership & Responsibility

- Using interpersonal and problem-solving skills to influence and guide others toward a goal

Objective

Students will be introduced to, understand, and practice the following concepts:

1. Surface Area of Rectangular Prisms and Cubes
2. Two-dimensional Nets of Three-dimensional figures
3. Simplifying Geometric Formulas

Materials

- Rulers – inches (Class Set)
- Several samples of real-life rectangular prisms of various sizes & dimensions:
    - tissue boxes, cereal boxes, other food and/or storage cardboard boxes, preferably enough for each table group to share.

      Starting with the letter "A", clearly label each prism on the outside with a large marker so that each prism is labeled with a different letter (A, B,C, D….etc.).

      You may want to re-use your boxes from Lesson 2.
- Scissors (1 pair for teacher & for each group of students)

  Note: If the student scissors are not strong enough to cut apart the sample boxes, the teacher may need to pre-cut all but one sample box ahead of time. Be sure to leave one box uncut for the direct instruction portion in Section 2 below.
- Day 3 Worksheet (KEY)—(see Lesson 3 lesson plans)
- Day 4 Worksheet (one for each student)
- Day 4 Worksheet (KEY)—Teacher must provide the answers for each prism
- Prism A, B, C, etc. based upon what exact prisms the teacher brought for the class to study.

Students bring:

- Completed Lesson 3 Worksheet (one for each student) from yesterday's class work/homework—(see Lesson 3 lesson plans)
- Rubik's Cube
- "You Can Do the Rubik's Cube" Solution Guide

Procedure

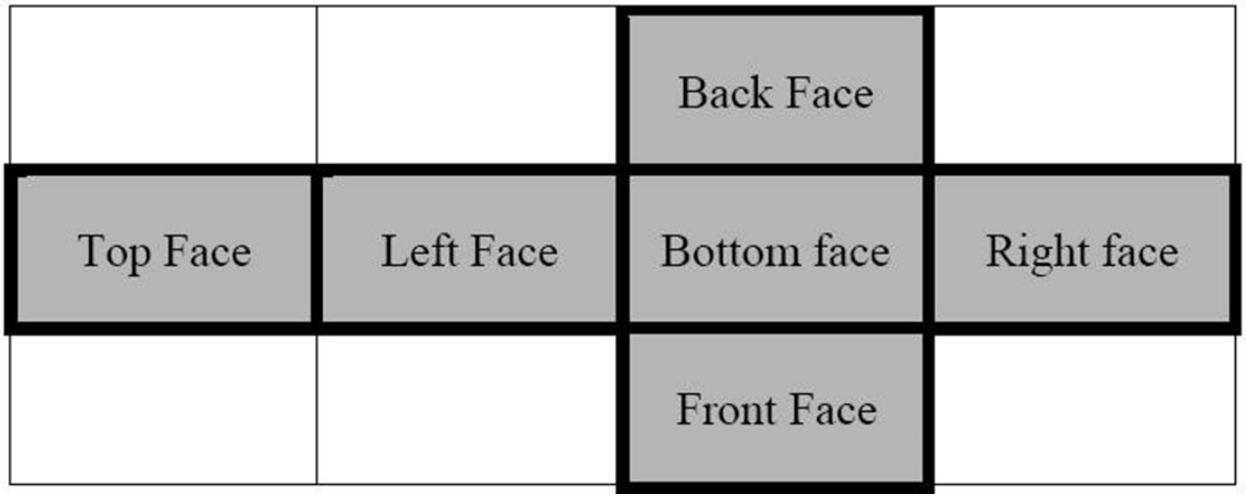
Introduction

1. Take out homework and discuss answers from the homework portion of the Day 3 Worksheet, sharing discoveries, observations, challenges, etc.
2. Take out Rubik's Cubes (most likely, several students were able to use their solution guides to complete Stages 4 & 5, and possibly more)
3. Inform the class that today they will become more familiar with their Rubik's Cubes while learning about *surface area* and how to simplify geometric formulas.

Procedure

Instruction

1. Pass out a copy of the Day 4 Worksheet to each student.
2. Explain the definition of Surface Area: The total area made up of all faces of a polygon. In the case of a rectangular prism, surface area is the total of the area of all six faces (top, bottom, front, back, left, and right faces).
3. As an analogy, explain that surface area can be thought of as the total area of gift-wrapping paper that you will need to wrap a gift. When you unwrap the gift and lay out flat all the non- overlapping paper, then the surface area of that gift is the total area of the wrapping paper!
4. Explain that similar to the wrapping paper example, polyhedrons (and specifically for today, rectangular prisms) can actually be laid flat to help determine their total surface area. To do this, we can take a rectangular prism and literally cut it up so that all six panels lay flat! We call this a net.
5. Take out one of the sample rectangular prisms from Day 2's lesson, and in front of the class use your scissors to cut it apart so that all six panels lay flat in the shape of a net:



6. Explain that the above net reveals that the rectangular prism (box) that you just cut apart is actually made up of six rectangles, each representing one of the six faces of the original rectangular prism (again, see diagram above).
7. Explain that if you simply find the area of each of the six rectangles and add them up, you will have found the total surface area of the rectangular prism (at this point, re-assemble the box so that the class understands you are actually finding information about a three-dimensional shape, although the net is two-dimensional).
8. With the class watching, use an inches ruler to measure each rectangle of the sample box you have cut into a net.

Procedure
Instruction (continued)

9. Together with the class, carefully complete section 1 of the Day 4 Worksheet (see below) by measuring the length and width of each face (rounded to the nearest inch), calculating the area of each face and the total surface area of the box. Remind students to record the same data you record on their own worksheet.
10. When done, point out to the class that there is a lot of repeated information on the worksheet. For example, both the top and bottom faces have the exact same data, while the left and right faces do too, as do the front and back faces.
11. Explain that since there is so much repeated data with rectangular prisms, there is a shortcut for calculating the surface area of a rectangular prism: just find the area of the top (or bottom) face, the left (or right) face, and the front (or back) face, and then double each of those three areas, then add together all three products.
12. In other words, the formula for the surface area of a rectangular prism is:
    Surface Area of a Rectangular Prism:
    $$\mathbf{2}[(\boldsymbol{L\ x\ W}) + (\boldsymbol{W\ x\ H}) + (\boldsymbol{L\ x\ H})]\ \boldsymbol{units^2}$$
13. Check for understanding.

Procedure
Guided Practice

1. Tell the class to turn their worksheets over to the blank back side to draw their own *net* of all
2. six faces of their Rubik's Cube, labeling each side to make sure that they have accounted for all six sides.
3. Then, tell the class to write in the dimensions, in *cubies*, of each face. Each face should have two different dimensions labeled: length (3 *cubies*) and width (3 *cubies*).
   (Note: See Lesson 1 lesson plan for explanation of the word "*cubie*", if necessary.)
4. Next, tell the class to calculate the area of each face $(3\ x\ 3\ =\ 9\ cubies^2)$ and to label the area of each face within each of the six faces on the net.
5. Finally, tell the class to add up all six faces on their net, resulting in the total *surface area*, in *cubies*, of a Rubik's Cube.
6. Ask students to transfer all of this information onto the front side of their Day 3 Worksheet, filling in the middle section of the worksheet.
7. Point out to the class the large amount of repetitive data in the Rubik's Cube portion of the chart.
8. Explain to the class that since each face of a cube is identical in size and shape, that a shortcut can be used to find the *surface area* of a cube: Find the area of one face $(length\ x\ length\ =\ L^2)$, and multiply that product by 6 to account for the area of all six identical sides. Therefore, the formula for finding the *surface area* of a cube is simply: Surface Area of a Cube = $6L^2\ units^2$

Procedure

Independent Practice

1. Give each group a labeled rectangular prism you have brought with you, preferably one for each table group or pair of students (see supplies list above).
2. Pass out an inch ruler to each student.
3. Ask the class to work together in groups to determine the dimensions, area of each face, and total surface area of the rectangular prism they have been assigned.
4. Tell the class to first begin this process by drawing a net of the prism on the back side of their worksheet, making sure to account for all six faces and also making sure to record the dimensions and square area of each face in inches.
5. After a few minutes, review answers for each prism with the class, beginning with Prism A.
6. After each group has shared, discuss any patterns discovered, interesting observations made, shortcut formulas used, etc.
7. Answer any questions & clarify any misunderstandings about the meaning and measurement of the area of faces, total surface area, shortcut formulas, and nets.

Procedure

Closure & Review

1. Review the definitions and formulas learned/reviewed today: net, face, area, surface area, and the two shortcut formulas for rectangular prisms and cubes.
2. Explain that once these concepts are well understood for rectangular prisms, they are much easier to understand and calculate for triangular prisms, pyramids, cylinders, and other complicated three-dimensional polygons and shapes.
3. Tell students to turn to Stage 6 (pp. 8—9) of their Solution Guides.
4. Briefly review these two pages with the students, making sure that they still know how to hold their cubes and how to rotate the faces of their Rubik's Cube according to the pictures provided.
5. Especially clarify that some of the sequences on these two pages may have to be repeated. Encourage students to repeat the instructions carefully in their Solution Guide.
6. Put your "expert cubers" who have shown a natural ability to understand the Solution Guide and communicate it to others to work. Discourage these helpers from "solving" their neighbors' cubes for them. Instead, encourage these helpers to explain and demonstrate by using their own cubes.
7. Assist as needed.

Procedure

Closure & Review

(continued)

8. In the last few minutes of class, assign the following homework:
   a. Re-read Solution Guide pages 8—9
   b. Complete the Rubik's Cube (Stages 1 through 6) and bring their Rubik's Cube AND Solution Guide with them to class the next day. Encourage students that it's OK if they can't solve Stage 6 by tomorrow. They will have more time after the unit to complete this goal. The important thing is that they are struggling and trying hard to grasp the Solution Guide instructions.
   c. Find one rectangular prism at home, draw and label a net for it, including each face's dimensions and area, and calculate the total surface area for that rectangular prism.
   d. Bring your rectangular prism from home (if possible) and your drawing of its net, dimensions, and overall calculations.
   e. Encourage students to use the shortcut formulas on their homework as appropriate.

Lesson 4
Introductory Geometry: Surface Area/ Nets

Name_____ **Date**_____

Worksheet

Surface Area of a Rectangular Prism = $2[(L\ x\ W)\ +\ (W\ x\ H)\ +\ (L\ x\ H)]\ units^2$

Surface Area of a Cube = $6L^2\ units^2$

Lesson 4
Introductory Geometry: Surface Area/ Nets

Name_____Date_____

Worksheet

Answer Key

Surface Area of a Rectangular Prism = $2[(L\ x\ W)\ +\ (W\ x\ H)\ +\ (L\ x\ H)]\ units^2$

Surface Area of a Cube = $6L^2\ units^2$

Rectangular Prism	Length (in.)	Width (in.)	L x W = Area (in^2)
Front Face	inches	inches	$inches^2$
Back Face	inches	inches	$inches^2$
Left Face	inches	inches	$inches^2$
Right Face	inches	inches	$inches^2$
Top Face	inches	inches	$inches^2$
Bottom Face	inches	inches	$inches^2$
Surface Area			$inches^2$
Rubik's Cube	**Length**	**Width**	**L x W= Area ($cubies^2$)**
Front Face	3 cubies	3 cubies	9 $cubies^2$
Back Face	3 cubies	3 cubies	9 $cubies^2$
Left Face	3 cubies	3 cubies	9 $cubies^2$
Right Face	3 cubies	3 cubies	9 $cubies^2$
Top Face	3 cubies	3 cubies	9 $cubies^2$
Bottom Face	3 cubies	3 cubies	9 $cubies^2$
Surface Area			54 $cubies^2$
Rectangular Prism 2	Length (in.)	Width (in.)	L x W = Area (in^2)
Front Face	inches	inches	$inches^2$
Back Face	inches	inches	$inches^2$
Left Face	inches	inches	$inches^2$
Right Face	inches	inches	$inches^2$
Top Face	inches	inches	$inches^2$
Bottom Face	inches	inches	$inches^2$
Surface Area			$inches^2$