<table>
<thead>
<tr>
<th>Kindergarten</th>
<th>Common Core</th>
<th>National</th>
</tr>
</thead>
<tbody>
<tr>
<td>K.G.1 - Names of shapes</td>
<td>Number and Operations</td>
<td>• Understand and represent common fractions</td>
</tr>
<tr>
<td>K.OA.5 - Add and subtract within 5</td>
<td>Algebra</td>
<td>• Sort, classify, and order objects by properties</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Recognize and describe patterns</td>
</tr>
<tr>
<td>Grade 1</td>
<td>1.G.1 - Defining attributes of shapes</td>
<td>Geometry</td>
</tr>
<tr>
<td></td>
<td>1.OA.4 - Subtraction as an unknown - addend problem</td>
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<td></td>
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<tr>
<td>Grade 2</td>
<td>2.MD.7 - Telling time</td>
<td>Data Analysis and Probability</td>
</tr>
<tr>
<td></td>
<td>2.OA.2 - Add and subtract within 20</td>
<td></td>
</tr>
<tr>
<td>Grade 3</td>
<td>3.MD.1 - Telling time</td>
<td></td>
</tr>
<tr>
<td>Grade 4</td>
<td>4.G.1 - Identify angles, perpendicular and parallel lines in two-dimensional figures</td>
<td>Geometry</td>
</tr>
<tr>
<td>Grade 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade 6</td>
<td>6.NS.3 - Use standard algorithms</td>
<td>Geometry</td>
</tr>
</tbody>
</table>
How to Solve The Rubik's Cube
Lesson 4 - The Middle Layer

Contents:
Overview & Objectives .................................................. 2
Whole Class Lesson .......................................................... 3
  Review - The White Corners, Lesson 3 .......... 3
  Lesson Vocabulary ...................................................... 4
  Lesson Focus ............................................................... 4
  Differentiation - Leveled Group Activities ...... 7
  Lesson Review ............................................................. 13
  Lesson Extension .......................................................... 14
  Trivia & Evaluation ..................................................... 15
  At Home Connection ................................................... 16
PowerPoint ...................................................................... 17

21st Century Learning Skills:
For complete details, see Standards & Skills Book

Learning & Innovation Skills:
• Creativity and Innovation
• Critical Thinking and Problem Solving
• Communication and Collaboration

Life & Career Skills:
• Flexibility and Adaptability
• Initiative and Self Direction
• Social and Cross-Cultural Skills
• Productivity and Accountability
• Leadership and Responsibility

Media Literacy:
• Information Literacy

About the Author:
Amber Baur has been an educator and math coach in Southern California since 2003. She is a certified GATE teacher with a Master of Education in Cross-Cultural Teaching.

Acknowledgments:
The "How to Solve the Rubik's Cube" lesson plans are intended as a comprehensive instructional guide for teachers and educators based on the You CAN Do The Rubik's Cube solution guide. We wish to thank all our friends in the Rubik's community for their support and inspiration.
OVERVIEW

This lesson will focus on solving the horizontal Middle Layer of the cube while keeping the WHITE cross and WHITE corners intact.

OBJECTIVES

By the end of the class period, students will be able to:

- Understand the algorithms used to solve the Middle Layer.
- Identify the direction of rotation an edge piece needs to move.
- Apply the knowledge of clockwise and counter-clockwise moves to manipulate the unit pieces of the cube.

MATERIALS

- Class set of Rubik’s Cubes and Solution Guides
- Crayons/Markers/Colored Pencils
- (Optional) Method for viewing PowerPoint
- (Optional) “The Middle Layer” PowerPoint file

SOLUTION GUIDE

This lesson correlates with **STAGE 4** of the *You CAN do the Rubik’s Cube Solution Guide.*

GOAL

**PP2**
Whole Class Lesson

Review from the White Corners

Each student should have a solved or scrambled Rubik’s Cube.

Review the ¼ turns

Tell students to practice the following ¼ turn moves with you:

"R" ¼ turn, "Ri" ¼ turn, "L" ¼ turn, "Li" ¼ turn, "U" ¼ turn, "Ui" ¼ turn, "D" ¼ turn, "Di" ¼ turn, "F" ¼ turn, "Fi" ¼ turn, "B" ¼ turn, and "Bi" ¼ turn.

Review - To get the White Corners matched with the correct faces:

Tell Students:

- Position a White corner piece on the bottom layer underneath its intended position.

- Use the algorithm as many times as needed until the corner is in the correct position.

- Repeat the steps for each White corner until all 4 corners are in the correct positions.
Lesson Vocabulary

**Horizontal**

Tell students:
- Horizontal is the name that describes when a line (or row) is parallel to the horizon. When looking at the cube, a horizontal line is parallel to the bottom or top layer of the cube. (Note to teacher: Check to see if students know the meaning of parallel.)

**Vertical**

Tell students:
- Vertical is the name that describes when a line (or row) is perpendicular to the horizon. When looking at the cube, a vertical line is parallel to the right or left layer of the cube. (Note to teacher: Check to see if students know the meaning of perpendicular.)

Lesson Focus

Each student should have a cube with the **White** cross and **White** corners solved.

After each instruction, walk around and check to be sure students have completed the instructions accurately.

**With the White face on the Down face and the Yellow face on the Up face:**

Tell students:
- Find an edge piece on the top layer that is not **Yellow** on the **Up** or **Front** face.

- Match the **Front** face of the edge to the center piece of the same color by twisting the layer until there is a vertical middle line (layer) of all one color on the **Front** face of the cube. In the example below, this is **Blue** but could be **Red**, **Orange** or **Green**.

- Look at the **Up** face of the edge:
  - If the **Up** face color belongs on the **Right** face of the cube:

- Use the following algorithm:
• If the **UP** face color belongs on the **LEFT** face of the cube:

![Diagram of solving a Rubik's Cube](image)

• Use the following algorithm:

![Algorithm steps](image)

---

**With the WHITE face on the DOWN face and the YELLOW face on the UP face:**

**Tell** students:

• Find another edge on the top layer that is not **YELLOW** on the **UP** or **FRONT** face.

• Match the **FRONT** face of the edge to the center piece of the same color by twisting the top layer until there is a vertical middle line of all one color on the **FRONT** face of the cube.

• Look at the **UP** face of the edge:
  • If the **UP** face color belongs on the **RIGHT** face of the cube:

![Algorithm steps for the RIGHT face](image)

• If the **UP** face color belongs on the **LEFT** face of the cube:

![Algorithm steps for the LEFT face](image)

**Tell** students:

• Check to make sure the **WHITE** cross and the **WHITE** corners on the **DOWN** face remain intact after you have repositioned an **UP** or **FRONT** face non-**YELLOW** edge.

• Repeat the previous instructions until the horizontal Middle Layer of each face has been solved.
TROUBLESHOOTING

- If there are no more non-YELLOW edges to work with on the top layer, use either one of the algorithms to place a YELLOW edge on the RIGHT or LEFT face where there is an unsolved edge. (Note to teachers: This will not solve for the Middle Layer, but will shift the edges around so there is a non-YELLOW edge on the top layer to work with.)

- By moving a YELLOW edge to an unsolved place in the Middle Layer, the non-YELLOW edges will move to give you the opportunity to work with a non-YELLOW edge.

- If you accidentally move an edge piece to the wrong place:
  - Leave it there and continue working with the other edge pieces.
  - Eventually, you will find the correct edge piece and place it in its correct position.
  - The misplaced edge piece will be moved and available to move to its correct place.

- If you move an UP face, edge piece clockwise, when it should have moved counter-clockwise (or counter-clockwise, when it should have moved clockwise), you can “undo” the move by inverting (doing the opposite) each ¼ turn and performing the algorithm inverted and backwards.

  - For example, if you moved the UP face, edge piece clockwise, you used the algorithm:

  - To “undo” the move, you would use the algorithm:

Ask students: Raise their when they have completed the Middle Layer on all faces. Check for accuracy.

GOAL

Tell students to examine their cube:

- To be sure the DOWN face has a completed WHITE face.
- To be sure the BOTTOM Layer and MIDDLE Layer of each face matches the center piece color.

CONGRATULATIONS!

THE MIDDLE LAYER HAS BEEN ACHIEVED!
DIFFERENTIATION – LEVELED GROUP ACTIVITIES

Based on your observations and background knowledge of students, divide students into small groups according to the groups below. Within each of the leveled groups, students may be further divided into pairs or mini-groups. The groups should be flexible. Students can move in and out of the leveled groups based on their understanding and mastery of activities within each level. Depending on the number of students, there may be multiple groups within each level. The lettered levels are meant to differentiate activities, not to maintain only four groups.

• **M** – **Modify** the lesson for understanding (Individual or Pairs Suggested)
  Use Appendix 4.M
  Students in this group should:
  • Need some assistance identifying edges and direction.
  • Be familiar with the **WHITE** corners.
  • Be familiar with the ¼ turns and letter representations.

• **A** – **Apply** the lesson to repeated practice (Groups or Pairs Suggested)
  Use Appendix 4.A
  Students in this group should:
  • Be able to solve the **WHITE** corners consistently.
  • Be able to work well with others.
  • Understand clockwise and counter-clockwise without help.

• **T** – **Re-Teach** the lesson for mastery (Pairs Suggested)
  Use Appendix 4.T
  Students in this group should:
  • Be familiar with clockwise and counter-clockwise movements.
  • Be able to identify edges.
  • Need more time to master The Middle Layer.

• **H** – **Higher** level learning for enrichment
  Use Appendix 4.H
  Students in this group should:
  • Have mastered the activities in Group A.
  • Be able to work independently.
This is the MIDDLE Layer goal: Use your crayons, markers, or pencils to color in the goal on the blank cube.

Have a teacher or friend help you find an edge on the UP face that is \textit{not} YELLOW on the UP or FRONT face.

Twist the UP face until the FRONT face of that edge matches the center piece. Should the UP face of the middle edge go to the \textbf{RIGHT} or to the \textbf{LEFT} (clockwise or counter-clockwise)?

\begin{align*}
\text{RIGHT} & \quad \text{LEFT}
\end{align*}
If the **UP** face should move to the **RIGHT** (clockwise) then do these 8 moves to put that piece on the **RIGHT** face:

If the **UP** face should move to the **LEFT** (counter-clockwise) then do these 8 moves to put that piece on the **LEFT** face:

Have a teacher or friend help you find another edge on the **UP** face, that is *not* **YELLOW** on the **UP** or **FRONT** face and complete the tasks above. Continue finding *non-YELLOW* edges and placing them correctly until the Middle Layer is completely solved.
Appendix 4.A

The Middle Layer

**YELLOW face UP**

- Find a *non-YELLOW* edge.
- Twist the **UP** face until the **FRONT** face of the *non-YELLOW* edge is in line with the matching center piece.

Should the **UP** face, middle edge move clockwise to the **RIGHT**?

![Diagram](image1)

Should the **UP** face, middle edge move counter-clockwise to the **LEFT**?

![Diagram](image2)

Find another *non-YELLOW* edge on the **UP** or **FRONT** face.

- Twist the **UP** face until the **FRONT** face of the *non-YELLOW* edge is in line with the matching center piece.
- Move the edge piece clockwise or counter-clockwise using one of the algorithms above until the edge piece is in the correct position on the Middle Layer.

Repeat the previous instructions until the Middle Layer has been completed.
The Middle Layer

Use the words in the box to fill in the blanks. Words may be used more than once. Not all words may be used.

<table>
<thead>
<tr>
<th>RIGHT</th>
<th>LEFT</th>
<th>RED</th>
<th>BLUE</th>
<th>WHITE</th>
</tr>
</thead>
<tbody>
<tr>
<td>UP</td>
<td>DOWN</td>
<td>ORANGE</td>
<td>GREEN</td>
<td>YELLOW</td>
</tr>
</tbody>
</table>

You may need multiple copies of this sheet to complete the Middle Layer.

With the **YELLOW** face **UP**:

1. Find an edge piece on the **TOP** layer that is not **YELLOW** on the **UP** or **FRONT** face.

2. Position the cube so the edge is facing towards you (facing front).
   - The **UP** face color of the edge is ____________________.
   - The **FRONT** face color of the edge is ____________________.

3. Twist the **UP** face so that the **FRONT** face color of the edge is in a vertical (up and down) line with the same color center piece.

4. Position the cube so the center piece and matching edge are facing front.
   - The **FRONT** face color of the edge is ____________________ and is in line with the center piece edge, which is the color ____________________. I have a vertical line all in the color__________.

5. Look at the **RIGHT** and **LEFT** faces.
   - The color on the **RIGHT** face is ____________________.
   - The color on the **LEFT** face is ____________________.
   - The color on the **UP** face of the middle edge is ____________________.
   - The **UP** face of the middle edge should be on the ____________________ face.
   - The **UP** face of the middle edge needs to move ____________________ in order to get to the ____________________ face.

6. Choose an algorithm to use to move the **UP** face middle edge.
   - To move the **UP** face of the middle edge ____________________ to the ____________________ face, I need to use the _____, _____, _____, _____, _____, _____, _____, _____ algorithm.
Undo the Middle Layer

Once you achieve the Middle Layer, try to undo it by inverting the algorithm and recreating the vertical middle line.

For example:

To get the **BLUE / RED** edge on the **RIGHT** face you would:

Now, try to "undo" the move, by first inverting all the moves, then doing them backwards:

Now, try solving for the Middle Layer, then "undoing" the Middle Layer entirely.

Use the rest of this sheet (and the back, if needed) to write down your inverted moves.
**Review**

Tell students:

- Find an edge on the **TOP** layer that is *not* **YELLOW** on the **UP** or **FRONT** face.
- Match the **FRONT** face of the edge to the center piece of the same color by twisting the **TOP** layer until there is a vertical middle line of all one color on the **FRONT** face of the cube.
- Look at the **UP** face of the edge:
  - If that color belongs on the **RIGHT** face (clockwise) of the cube:
    - Check to make sure the **WHITE** cross and the **WHITE** corners on the **DOWN** face remain intact after you have repositioned an **UP** or **FRONT** face non-**YELLOW** edge.
  - If that color belongs on the **LEFT** face (counter-clockwise) of the cube:
    - Repeat the instructions until the horizontal **MIDDLE** Layer of each face has been solved.
**Lesson Extension**

**Tell** students:

- To "undo" a move on the Rubik's Cube, you do the opposite of what was done.
- In math, the same idea applies.
- To do the opposite of addition, you subtract.
- To do the opposite of subtraction, you add.
- For example:
  - If you have 2 Rubik's Cubes and you add 3 more, you will have 5 Rubik's Cubes all together.
  - If you subtract, or take away 3 Rubik's Cubes, you will be left with the original 2 cubes.

![Image of Rubik's Cubes showing addition and subtraction](image)

- The opposite of a positive is a negative.
- For example:
  - When using integers on a number line, adding opposites together will result in 0.
  - By adding +4 and -4, the result is 0.
  - On a number line:
    - Start at 0.
    - Go forward 1, 2, 3, 4 positive places.
    - Then go backward 1, 2, 3, 4 negative places.
    - The result is 0.
    - Opposite movements cancel each other out.

![Image of number line showing addition and subtraction](image)

- Another example:
  - \[ x + 5 = 11 \]
  - \[ x = 6 \]
  - By subtracting 5 from both sides of the equal sign, the problem was balanced and solved.
  - To get \( x \) alone, the opposite operation was used.
  - The opposite of +5 is -5.

**Tell** students:

- When the Bottom and Middle layers are completed, the cube is two-thirds solved.
- When the **WHITE** face is **DOWN** face and the **YELLOW** face is the **UP** face, there are three horizontal layers - Top, Middle, Bottom.
- Each layer has 12 squares around.
- Total 3 layers x 12 squares equals 36 squares around.
- If two layers are complete, then 24 squares are complete.
- 24 out of 36 can be simplified to 2 out of 3.
**Rubik’s® Trivia**

**Question:** Who was the inventor of the Rubik's Cube? Where was he from?

**Answer:** Erno Rubik; Hungary

**Evaluation**

Each student should be able to:

- Solve the Middle Layer of the Rubik's Cube.
- Identify direction of rotation a piece needs to move to be placed in its correct position.
- Apply the previous concepts of clockwise and counter-clockwise movements to directions of rotation.
- Understand the difference between vertical and horizontal lines.
Application to Math

- To “undo” a move on the Rubik’s Cube, you do the opposite of what was done.
- In math, the same idea applies.
- To do the opposite of addition, you subtract.
- To do the opposite of subtraction, you add.
- For example:
  - If you have 2 Rubik’s Cubes and you add 3 more, you will have 5 Rubik’s Cubes all together.
  - If you subtract, or take away 3 Rubik’s Cubes, you will be left with the original 2 cubes.
- The opposite of a positive is a negative.
- For example:
  - When using integers on a number line, adding opposites together will result in 0.
  - By adding +4 and -4, the result is 0.
- On a number line:
  - Start at 0.
  - Go forward 1, 2, 3, 4 positive places.
  - Then go backward 1, 2, 3, 4 negative places.
  - The result is 0.
  - Opposite movements cancel each other out.

WHAT WE LEARNED

Solve the Middle Layer

Identify direction of rotation a piece needs to move

Apply clockwise and counter-clockwise movements to manipulate the cube

GOAL

REVIEW

- Find an edge on the UP face that is not YELLOW on the UP or FRONT face.
- Match the FRONT face of the edge to the center piece of the same color so that there is a vertical middle line of all one color on the FRONT face of the cube.
- Look at the UP face of the edge:
  - If that color belongs on the RIGHT face of the cube:
  - If that color belongs on the LEFT face of the cube:
Lesson 4

The MIDDLE Layer

PowerPoint
GOAL: The Middle Layer

The goal of this stage is to solve the Middle Layer while keeping the WHITE face intact (the WHITE cross and WHITE corners).
Inverted means opposite. By inverting a move, the move can be undone.

\[
\begin{align*}
\text{R} & \rightarrow \text{L} \\
\text{Ri} & \rightarrow \text{Li} \\
\text{D} & \rightarrow \text{Ui} \\
\text{B} & \rightarrow \text{Bi} \\
\text{Fi} & \rightarrow \text{Fi}
\end{align*}
\]
REVIEW

To get the WHITE corners matched with the correct faces:

• Position a WHITE corner on the BOTTOM layer underneath its intended position:

• Use the algorithm as many times as needed until the corner is correctly placed.

• Repeat the steps for each WHITE corner until all four corners are in the correct places.
Horizontal is the name that describes when a line (or row) is parallel to the horizon. When looking at the cube, a horizontal line is parallel to the BOTTOM or TOP layer of the cube.
VERTICAL

Vertical is the name that describes when a line (or row) is perpendicular to the horizon. When looking at the cube, a vertical line is parallel to the **RIGHT** or **LEFT** layer of the cube.

THE BLUE MIDDLE ROW IS VERTICAL
With the **WHITE** face on the **DOWN** face and the **YELLOW** face on the **UP** face:

- Find an edge on the **TOP** layer that is NOT **YELLOW** on the **UP** or **FRONT** face.
- Match the **FRONT** face of the edge to the center piece of the same color by twisting the **TOP** layer until there is a vertical middle line of all one color on the **FRONT** face of the cube.
- Look at the **UP** face of the edge.
- Decide whether the edge needs to move clockwise or counter-clockwise:
With the **WHITE** face on the **DOWN** face and the **YELLOW** face on the **UP** face:

- If the **UP** face color belongs on the **RIGHT** face of the cube

- If the **UP** face color belongs on the **LEFT** face of the cube
With the **WHITE** face on the **DOWN** face and the **YELLOW** face on the **UP** face:

- Find another edge on the **TOP** layer that is **NOT YELLOW** on the **UP** or **FRONT** face.
- Match the **FRONT** face of the edge to the center piece of the same color by twisting the **TOP** layer until that there is a vertical middle line of all one color on the **FRONT** face of the cube.
- Look at the **UP** face of the edge:

  - **If the UP face color needs to move clock-wise to the RIGHT face of the cube to its correct position:**
  
  ![Clock-wise moves](image1)

  - **If the UP face color needs to move counter-clockwise to the LEFT face of the cube to its correct position:**
  
  ![Counter-clockwise moves](image2)
If there are no more non-

YELLOW edges to work with, use one of the algorithms to place a

YELLOW edge on the RIGHT or LEFT face where there is an unsolved edge.

- By moving a YELLOW edge to an unsolved place in the middle layer, the non-

YELLOW edges will shift to give you the opportunity to work with a non-

YELLOW edge.
If you accidentally move an edge piece to the wrong place:

- Leave it there and continue working with the other edge pieces.
- Eventually, you will find the correct edge piece and place it in its correct place.
- The misplaced edge piece will be moved and available to move to its correct place.
You can “undo” the move by inverting (doing the opposite) each ¼ turn and performing the algorithm inverted and backwards.

- For example, if you moved the UP face, edge piece clockwise, you used the algorithm:

  - To “undo” the move, you would invert the moves and reverse the algorithm:
GOAL: The Middle Layer

The goal of this stage is to solve the Middle Layer while keeping the WHITE face intact (The WHITE cross and WHITE corners)

Examine your Rubik’s Cube

Does your Rubik’s Cube look like this?
Congratulations! Congratulations! Congratulations! Congratulations! Congratulations! Congratulations! Congratulations! Congratulations! Congratulations! Congratulations!

You have achieved

The Middle Layer
• Find an edge on the TOP layer that is NOT **YELLOW** on the UP or FRONT face.

• Match the FRONT face of the edge to the center piece of same color by twisting the TOP layer until there is a vertical middle line of all one color on the FRONT face of the cube.

• Look at the UP face of the edge:
  
  • If UP face color belongs on the RIGHT face (clockwise) of the cube:

  ![Diagram showing edge manipulation](image1)

  - Repeat the instruction until the horizontal middle line of each face has been solved.

  • Check to make sure the **WHITE** cross and the **WHITE** corners on the DOWN face remain intact after you have repositioned an UP or FRONT face non-YELLOW edge.

  ![Diagram showing edge manipulation](image2)
To “undo” a move on the Rubik’s Cube, you do the opposite of what was done.

In math, the same idea applies.

To do the opposite of addition, you subtract.

To do the opposite of subtraction, you add.

For example:

- If you have 2 Rubik’s Cubes and you add 3 more, you will have 5 Rubik’s Cubes all together.

- If you subtract, or take away 3 Rubik’s Cubes, you will be left with the original 2 cubes.

Lesson Extension

How does this lesson apply to math?
Lesson Extension

How does this lesson apply to math?

• For example:
  • The opposite of a positive is a negative.
  • When using integers on a number line, adding opposites together will result in 0.
  • By adding +4 and -4, the result is 0.
For example:

- By subtracting 5 from both sides of the equal sign, the problem was balanced and solved.
- To get x alone, the opposite operation was used.
- The opposite of +5 is -5.

\[
x + 5 = 11
\]

\[
\begin{array}{c}
-5 \\
\hline \\
6
\end{array}
\]

How does this lesson apply to math?
• When the **BOTTOM** and **MIDDLE** layers are complete, the cube is two-thirds solved.

• When the **WHITE** face is the **DOWN** face and the **YELLOW** face is the **UP** face, there are three horizontal layers – **TOP**, **MIDDLE**, **BOTTOM**.

• Each layer has 12 squares around.

• Total 3 layers x 12 squares = 36 squares around.

• If two layers are complete, then 24 squares are complete.

• 24 over 36 can be simplified to 2 over 3.
Question: Who was the inventor of the Rubik’s Cube?

Answer: Erno Rubik; from Hungary

Where was he from?
The Middle Layer

Quarter Turn - Reference Sheet

R  Ri  L  Li

D  Di  U  Ui

B  Bi  F  Fi