Middle School

The Mathematics of the Rubik's Cube

National Standards

Instructional programs from prekindergarten through grade 12 should enable all students to:

Understand numbers, ways of representing numbers, relationships among numbers, and number systems

Grades 6–8 Expectations: In grades 6–8 all students should–

develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation;

use factors, multiples, prime factorization, and relatively prime numbers to solve problems;

Texas Essential Knowledge & Skills

Math 8.2 A-C Number and operations

The student applies mathematical process standards to represent and use real numbers in a variety of forms. The student is expected to

(A) extend previous knowledge of sets and subsets using a visual representation to describe relationships between sets of real numbers;

(B) approximate the value of an irrational number, including π and square roots of numbers less than 225, and locate that rational number approximation on a number line;

(C) convert between standard decimal notation and scientific notation;

Materials

- One Rubik's Cube per student, or per small group of students
- Student Handout (copy one for each student)
- Calculators, one per student or per small group of students

Objective

In this lesson, students will explore the possible number ways the pieces of a Rubik's Cube can be arranged, and still fit the criteria for a Rubik's Cube. Colors are oriented in a set way, so some pieces (such as a yellow-white edge) would not exist on a Rubik's Cube. Students will find all possible combinations for a 6-colored 3x3 cube, and then divide to take out the 'impossible pieces.

Lesson Outline

Once students have the appropriate materials, they can read the student handout and follow the directions.

©1974 Rubik's® Used under license Rubik's Brand Ltd. All rights reserved.
STEM.orgTM is a trademark of Initiative Science,LLC.and is used with permission

The Mathematics of the Rubik's Cube

I. Possible Number of Combinations of the Corner Cubes

- There are a total of _____ corner cubes that can be in _____ different locations around the cube at any given time. We can choose a location in 8! possible ways. $(8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)$ Since the corner cube has _____ different colors, it can be rotated in _____ different variations. So there are 3^8 different orientations of a corner cubes altogether.

  The total possible combinations of the corner cubes is $(8!)(3^8)$. This number equals _____.

II. Possible Number of Combinations of the Edge Cubes

- There are a total of _____ edge cubes that can be in _____ different locations around the cube at any given time. We can choose a location in _____! possible ways. Since the edge cube has _____ different colors, it can be rotated in _____ different variations. So there are $\_\_\_\_^{12}$ different orientations of the edge cubes altogether. The total possible combinations of the edge cubes are $(\_\_\_\_!)(\_\_\_\_^{12})$. This number equals _____.

III. Total Number of Combinations of the Entire Cube.

- To get our final answer, we simply multiply the two numbers we found above however, that would be over-counting. There is a small catch.
  - It turns out that you can't turn the faces in such a way as to switch the positions of two cubes while returning all the others to their original positions. Therefore if you get all but 2 cubes in place, there is only one attainable choice for the positions of the remaining 2 cubes. We must <u>divide the product we found above by 2</u>. Also…
  - If you are solving the corner pieces, the very last corner piece can only have 1 correct orientation so you must <u>divide the product above by 3</u> to cancel out the other possible variations of that corner piece. And…..
  - If you are solving the edge pieces, the very last piece can have only 1 correct orientation so you must <u>divide the product above by 2</u> to cancel out the other possible variations of that edge piece.

- Okay… we are ready for our final calculation.

$$\text{Total Number of Possibilities} = \frac{(8!)(3^8)(12!)(2^{12})}{(2)(3)(2)}$$

Write this number in scientific notation: _____

The full number in standard form is: _____

This number in word form would be: _____

_____

©1974 Rubik's® Used under license Rubik's Brand Ltd. All rights reserved.
www.youcandothecube.com

The Mathematics of the Rubik's Cube

By now you have figured out that there are 43 quintillion $\left(4.3\text{x}10^{19}\right)$ possible combinations of the cube. The question that is often asked is whether it is possible for somebody to come up with the solution to the cube by making random twists and turns. What is the probability of that happening?

When it comes to solving the Rubik's cube by a random series of twists and turns, you have a better chance of….

- Bowling a perfect game (300) sometime in your life (1 in 11,500)
- Becoming a professional athlete (1 in 22,000)
- Being struck by lightning today. (1 in 576,000)
- Spotting a U.F.O. today (1 in 3,000,000)
- Getting hit in the head from falling parts off of an airplane in your lifetime (1 in 10,000,000)
- A meteor landing on your house today (1 in 182,138,880,000,000)

Now, if you wanted to see all $4.3\text{x}10^{19}$ *different possible combinations of the cube how long would it take you to do the twists? Let's assume that you could perform three twists per second.*

- To determine the total number of seconds, you would have to divide $4.3\text{x}10^{19}$ by 3.
  _____seconds
- To determine the number of minutes, divide that number by 60.
  _____ minutes
- To determine the number of hours, divide that number by 60.
  _____ hours
- To determine the number of days, divide that number by 24.
  _____days
- To determine the number of years, divide that number by 365
  _____ years

Interesting Fact:
According to scientists, the universe is $1.37\text{x}10^{10}$ *years old.*

©1974 Rubik's® Used under license Rubik's Brand Ltd. All rights reserved.
www.youcandothecube.com

The Mathematics of the Rubik's Cube

ANSWER KEY

I. Possible Number of Combinations of the Corner Cubes

- There are a total of **8** corner cubes that can be in **8** different locations around the cube at any given time. We can choose a location in 8! possible ways. $(8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)$ Since the corner cube has **3** different colors, it can be rotated in **8** different variations. So there are 3^8 different orientations of a corner cubes altogether. The total possible combinations of the corner cubes is $(8!)(3^8)$. This number equals **264,539,520**.

II. Possible Number of Combinations of the Edge Cubes

- There are a total of **12** edge cubes that can be in **12** different locations around the cube at any given time. We can choose a location in **12 !** possible ways. Since the edge cube has **2** different colors, it can be rotated in **12** different variations. So there are **2^{12}** different orientations of the edge cubes altogether. The total possible combinations of the edge cubes are **(12!)** **(2^{12})**. This number equals **1,961,990,553,600**.

III. Total Number of Combinations of the Entire Cube.

- To get our final answer, we simply multiply the two numbers we found above however, that would be over-counting. There is a small catch.
  - It turns out that you can't turn the faces in such a way as to switch the positions of two cubes while returning all the others to their original positions. Therefore if you get all but 2 cubes in place, there is only one attainable choice for the positions of the remaining 2 cubes. We must divide the product we found above by 2. Also…
  - If you are solving the corner pieces, the very last corner piece can only have 1 correct orientation so you must divide the product above by 3 to cancel out the other possible variations of that corner piece. And…..
  - If you are solving the edge pieces, the very last piece can have only 1 correct orientation so you must divide the product above by 2 to cancel out the other possible variations of that edge piece.

- Okay… we are ready for our final calculation.

$$\text{Total Number of Possibilities} = \frac{(8!)(3^8)(12!)(2^{12})}{(2)(3)(2)}$$

Write this number in scientific notation: **4.3×10^{19}**

The full number is: **43,252,003,274,489,596,000**

This number in word form would be ***forty-three quintillion*, two hundred fifty-two quadrillion, three trillion, two hundred seventy-four billion, four hundred eighty-nine million, five hundred ninety-six thousand.**

©1974 Rubik's® Used under license Rubik's Brand Ltd. All rights reserved.
www.youcandothecube.com

The Mathematics of the Rubik's Cube

By now you have figured out that there are 43 quintillion $\left(4.3 \text{x} 10^{19}\right)$possible combinations of the cube. The question that is often asked is whether it is possible for somebody to come up with the solution to the cube by making random twists and turns.
What is the probability of that happening?

When it comes to solving the Rubik's cube by a random series of twists and turns, you have a better chance of….

- Bowling a perfect game (300) sometime in your life (1 in 11,500)
- Becoming a professional athlete (1 in 22,000)
- Being struck by lightning today (1 in 576,000)
- Spotting a U.F.O. today (1 in 3,000,000)
- Getting hit in the head from falling parts off of an airplane in your lifetime (1 in 10,000,000)
- A meteor landing on your house today (1 in 182,138,880,000,000)

Now, if you wanted to see all $4.3 \text{x} 10^{19}$ *different possible combinations of the cube how long would it take you to do the twists? Let's assume that you could perform three twists per second.*

- To determine the total number of seconds, you would have to divide $4.3 \text{x} 10^{19}$ by 3.
  $1.44 \text{ X } 10^{19}$ seconds
- To determine the number of minutes, divide that number by 60.
  $2.40 \text{ X } 10^{17}$ minutes
- To determine the number of hours, divide that number by 60.
  $4.00 \text{ X } 10^{15}$ hours
- To determine the number of days, divide that number by 24.
  $1.67 \text{ X } 10^{14}$ days
- To determine the number of years, divide that number by 365
  $4.57 \text{ X } 10^{11}$ years

©1974 Rubik's® Used under license Rubik's Brand Ltd. All rights reserved.
www.youcandothecube.com