Middle School

Writing Algorithms / Intro to Speed Solving

Texas Essential Knowledge & Skills

Mathematics 6.9A, 6.10A-B - Expressions, equations, and relationships.
The student applies mathematical process standards to use equations and inequalities to represent situations. The student is expected to:

write corresponding real-world problems given one-variable, one-step equations or inequalities.

model and solve one-variable, one-step equations and inequalities that represent problems, including geometric concepts; and

determine if the given value(s) make(s) one-variable, one-step equations or inequalities true.

Mathematics 8.3C - Proportionality.
The student applies mathematical process standards to use proportional relationships to describe dilations. The student is expected to:

use an algebraic representation to explain the effect of a given positive rational scale factor applied to two-dimensional figures on a coordinate plane with the origin as the center of dilation

Mathematics 8.10A Two-dimensional shapes.
The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:

generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane;

Objectives:

1) Students will learn about the cycles of algorithms.

2) Students will learn to rewrite algorithms by applying the Algebraic idea of inverse operations.

3) Students will learn to rewrite algorithms by applying the Geometric idea of mirror/reflective operations.

Materials:

Rubik's® Cubes (1 per student)
Access to mirrors (optional)
Intro to Speed Solving worksheet
Writing Inverse Algorithms worksheet
Writing Mirrored Algorithms worksheet
Exploring Advanced Speed Solving Algorithms
Internet access (for students)

Procedure:

1) Students should start with a solved Rubik's Cube. Hand out the Intro to Speed Solving worksheet.

2) Read through part 1 with the class. Have students test examples 1 & 2 to verify those algorithms' cycles. *Sadly, if a student makes a mistake during a cycle, they will have to resolve their cube and start over.

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3) In part 2, students are going to determine the cycle values of 3 more algorithms.

4) In part 3, students will make up their own random algorithm and test for its cycle value. *Some cycles values can be rather high, so make sure students stick with a 3s 5 turn algorithm.

5) Give out the Writing Inverse Algorithms worksheet and read through the intro together and walk through the example together.

6) Students will then write three inverse algorithms, test them, and describe their benefits when solving the Rubik's Cube.

7) When you get to the Writing Mirrored Algorithms worksheet, you can either have all the students pair up and do the mirror simulation, or you can take two volunteers and have them do the simulation in front of the class while the rest do the observations.

8) After going through the intro with the class, have the students write the three algorithms. This is where the optional mirrors could come in. I have had students perform the algorithms for #1s   3 while observing the moves of their reflection. It is challenging, but is a really good way for students to experience reflections.

9) Students should then test their newly written algorithms and describe their benefits when solving the Rubik's Cube.

10) With the Exploring Advanced Speed Solving Algorithms worksheet, have students start by looking up the answers to #1 & 2 online.

11) Read through the new notations together. Have students get out their Rubik's Cube, solve layers 1 & 2, and then explore and attempt using OLL and PLL.

Notes to Teacher: You will want to verify that your school internet does not block the websites needed for this activity.

Examples of mirrored algorithms can be clearly seen in the steps used to solve the middle layer of the Rubik's Cube. The directions for moving a piece to the right edge mirror those that move a piece to the left edge.

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Answer Key

Intro to Speed Solving

Part 1:

The algorithms that we use have cycles. This means that if you do the same algorithm over and over again, the puzzle will eventually go back to the state it started in prior to the repeated steps.

Examples:

Layer 3 – Permute the Cross (U R U R' U R U2 R' U) has a cycle of 3.

Layer 1 – Orient the Cross (R U' R' U R) has a cycle of 12.

Layer 2 algorithm U R U' R' U' F' U F has a cycle of 15.

Starting with a solved Rubik's Cube, test examples 1 & 2 by repeating each given algorithm the specified number of times. When you are done , the puzzle should be returned to the solved state.

Part 2:

Determine the cycle values of the following algorithms:

1) Layer 1 – Solving the Corners (R' D' R D) has a cycle of __6__.
2) Layer 3 – Orienting the Cross (F R U R' U' F') has a cycle of __6__.
3) Layer 3 – Permute the Corners (U R U' L' U R' U' L) has a cycle of __3__.

Part 3:

Make up your own algorithm consisting of 3-5 turns. The algorithm can be completely random; it does not need to contribute to solving the cube. Once you have made your algorithm, test it to determine its cycle value.

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Answer Key

Writing Inverse Algorithms

Inverse operations can be seen in writing "return" directions off of a map, or solving an algebraic equation (as seen below).

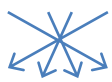
Looking at the equation $5 = \frac{3x+2}{4}$

Following the order of operations	Following inverse operations
Start at *x*	End at what *x* equals
1) multiply by 3	3) divide by 3
2) add 2	2) subtract 2
3) divide by 4	1) multiply by 4
Get an answer of 5	Start at the answer 5

Inverse operations have us "undo" everything that has been done. In other words, inverse operations make us do the opposite of each step AND in the reverse order.

Here is an example (this is not one of the learned algorithms):

Original algorithm: L, U', R', U



Inverse algorithm: U', R, U, L'

1) Layer 1/3 – Solving the Corners: R' D' R D

   Write the inverse algorithm: D', R', D, R

2) Layer 3 – Permute the Cross: U R U R' U R U2 R' U

   Write the inverse algorithm: U', R, U2, R', U', R, U', R', U'

3) Layer 3 – Permute the Corners: U R U' L' U R' U' L

   Write the inverse algorithm: L', U, R, U', L, U, R', U'

4) When done, compare answers with a neighbor. Then test out your new algorithms.
5) When will these algorithms be beneficial?

These algorithms reverse the order of a cycle. For example, if I was trying to solve the corner piece shown, I would R' D' R D once. However, if that corner piece was in the same place, but white tile was on the front face, that same algorithm would need to be used 5 times (however its inverse would only be needed once).

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Answer Key

Writing Mirrored Algorithms

Mirrored (or reflective) operations can be seen by observing movements in a mirror. Let's compile some observations by simulating mirrors. Find a partner and stand facing each other. Indicate who will be the "model" and who will be the mirror. The model will perform a couple of movements, and the partner will act as if they were the reflection in the mirror.

Movements by model	Movements by mirror
Wave with left hand	What hand is Mirror waving with?
Pat your head w/right hand	Is Mirror patting head? With what hand?
Put hands behind your back	Where are Mirror's hands?
Slowly rotate right arm clockwise	Which direction is Mirror's arm rotating?
Turn body right, rotating 90° clockwise	Which way did Mirror rotate?
Lower your chin, then raise it	When did Mirror's chin go down? Up?

What we hopefully noticed, was that with mirrored operations, up is still up, down is still down, front is still front, and back is still back. We may have also noticed that right becomes left, left becomes right, clockwise becomes counterclockwise, and counterclockwise becomes clockwise.

Here is an example:

Original algorithm: L, U', R', U

↓ ↓ ↓ ↓

Mirrored algorithm: R', U, L, U'

1) Layer 1/3 – Solving the Corners: **R' D' R D**

   Write the mirrored algorithm: *L, D, L', D'*

2) Layer 3 – Permute the Cross: **U R U R' U R U2 R' U**

   Write the mirrored algorithm: *U', L', U', L, U', L', U2, L, U'*

3) Layer 3 – Permute the Corners: **U R U' L' U R' U' L**

   Write the mirrored algorithm: *U', L', U, R, U', L, U, R'*

4) When done, compare answers with a neighbor. Then test out your new algorithms.
5) When will these algorithms be beneficial?

Again, these algorithms reverse the order of the cycles, but with a couple of difference: First, pieces that you would line up on the right prior to solving will now need to be lined up on the left (and vice versa); and second, algorithms that were originally right-hand dependent will now be left-hand dependent (and vice versa).

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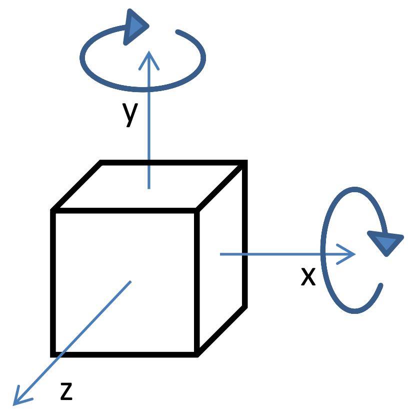
Answer Key

Exploring Advanced Speed Solving Algorithms

1) What does acronym OLL stand for? What does OLL mean? (search the internet)
   Orient Last Layer: to get all remaining pieces turned the right way
2) What does acronym PLL stand for? What does PLL mean? (search the internet)
   Permute Last Layer: to get all remaining pieces moved to correct locations

In order to use them, we will need more abbreviated notations. Here are the notations we have used and some of the new letters/sets that may come up in more complex algorithms:

- F (front) – the side facing toward, as viewed by the solver
- B (back) – the side that is opposite the front, as viewed by the solver
- L (left) – the side to the left of the front, as viewed by the solver
- R (right) – the side to the right of the front, as viewed by the solver
- U (up) - the side on top, as viewed by the solver
- D (down) – the side on bottom, as viewed by the solver
  *The six letters above assume a 90° clockwise rotation.
- 2 (two) – turn the given face twice
- ‘ (apostrophe) – turn counterclockwise
- f (front two faces)
- b (back two faces)
- l (left two faces)
- r (right two faces)
- u (upper two faces)
- d (downward two faces)
- x (rotate entire cube) – D will become F
- y (rotate entire cube) – R will become F
- z (rotate entire cube) – U will become R



- Solve layers 1 and 2 of a scrambled Rubik’s Cube, but not the last layer. Then go to one of the following sites (top preferred):
  - http://www.cubezone.be/oll.html
  - https://ruwix.com/the-rubiks-cube/rubiks-cube-solution-with-advanced-friedrich-method-tutorial/orient-the-last-layer-oll/
  - https://www.speedsolving.com/wiki/index.php/OLL

  Match up your mixed-up third layer to the same scenario from the site’s list. Then follow the provided algorithm to orient the last layer.

3) When successful in part 3, go to one of the following sites (top preferred):
   - http://www.cubezone.be/pll.html
   - https://ruwix.com/the-rubiks-cube/rubiks-cube-solution-with-advanced-friedrich-method-tutorial/permutate-the-last-layer-pll/
   - https://www.speedsolving.com/wiki/index.php/PLL

   Match your puzzle to the same scenario on the site and follow the corresponding algorithm.

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Get an answer of 5

<u>Following inverse operations</u>
End at what x equals
3) divide by 3
2) subtract 2
1) multiply by 4
Start at the answer 5

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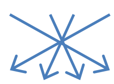

Inverse algorithm: U', R, U, L'

1) Layer 1/3 – Solving the Corners: R' D' R D

   Write the inverse algorithm:

2) Layer 3 – Permute the Cross: U R U R' U R U2 R' U

   Write the inverse algorithm:

3) Layer 3 – Permute the Corners: U R U' L' U R' U' L

   Write the inverse algorithm:

4) When done, compare answers with a neighbor. Then test out your new algorithms.

5) When will these algorithms be beneficial?

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Original algorithm: L, U', R', U

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Mirrored algorithm: R', U, L, U'

1) Layer 1/3 – Solving the Corners: R' D' R D

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2) Layer 3 – Permute the Cross: U R U R' U R U2 R' U

   Write the mirrored algorithm:

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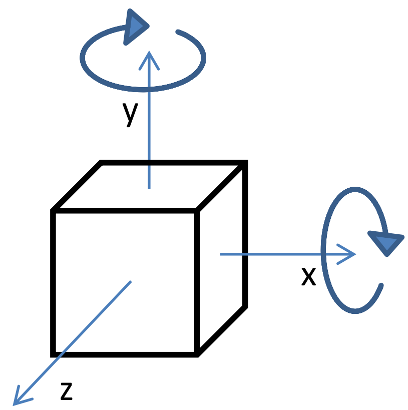

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